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Accurate carrier-removal technique based on zero padding in Fourier transform method for carrier interferogram analysis

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ABSTRACT

Based on interferogram zero padding and fast Fourier transform (FFT) methods, an effective, straightforward and stable carrier-removal approach in Fourier transform (FT) based method for carrier interferogram analysis is proposed. The spatial carrier interferogram is firstly extrapolated by zero padding method, and the carrier-frequency values within a small fraction of an integral (or a pixel) are estimated from the extrapolation interferogram with FFT method. Then the carrier-phase component is removed by subtracting a pure carrier-frequency phase constructed by the estimated carrier-frequencies in the spatial domain. Numerical simulations and experiments are given to demonstrate the performance of the proposed method and the results show that the proposed method is effective and stable for suppressing the carrier-removal error in the FT method for carrier interferogram analysis.

1. Introduction

The Fourier transform based (FT) method is one of the most popular interference interferogram analysis methods for high accuracy and automatic phase measurement systems. The FT method was originally proposed and demonstrated by Takeda et al. [1], and then many works were published afterward [2–8]. Comparing to temporal phase-shifting (PS) method, the FT method usually requires only one interferogram, which makes it is less sensitive to circumstantial disturbances and vibrations [9]. Therefore, it has been widespread applied to various kinds of interferometric applications, such as optical interferometer measurements [10–12], and holography interferometry and its applications in phase microscopy [13–16].

The major idea of the FT method can be described as follows [1–4,8]. The deformed fringe pattern $g(x,y)$ with linear-carrier is generally expressed as

$$g(x,y)=a(x,y)+b(x,y)\cos\left[2\pi(f_{0x}x+f_{0y}y)+\phi(x,y)\right] \tag{1}$$

where $a(x,y)$, $b(x,y)$ are the background illumination and the modulation intensities, respectively; f_{0x} and f_{0y} are the introduced spatial carrier-frequencies along x and y directions, respectively; $\phi(x,y)$ is the modulating phase. The carrier interferogram, $cos(2\pi f_{0x}x+f_{0y}y)$, serves as an carrier information for recording the measured phase data but it will simultaneously introduce a carrier phase component, $2\pi(f_{0x}x+f_{0y}y)$, in the phase extraction procedure [7]. Hence the carrier phase component must be subtracted or removed from the overall phase distribution for evaluation of the phase of the measured phase component $\phi(x,y)$. And it is generally achieved using the traditional FT method, as follows.

By using the Euler formula to expand the cosine term in Eq. (1), we have

$$g(x,y)=a(x,y)+c(x,y)\exp\left[j2\pi(f_{0x}x+f_{0y}y)\right] + c^{*}(x,y)\exp\left[-j2\pi(f_{0x}x+f_{0y}y)\right] \tag{2}$$

with the definition

$$c(x,y)=\frac{1}{2}b(x,y)\exp\left[j\phi(x,y)\right] \tag{3}$$

and superscript * denotes the complex conjugate. Taking the Fourier transform in two-dimension for the carrier interferogram expressed in Eq. (2), we have

$$G(f_x,f_y)=A(f_x,f_y)+C(f_x-f_{0x},f_y-f_{0y})+C^{*}(f_x+f_{0x},f_y+f_{0y}) \tag{4}$$

where f_x and f_y are the spatial frequency coordinates in the frequency domain. $G(f_x,f_y)$, $A(f_x,f_y)$, $C(f_x,f_y)$ and $C^{*}(f_x,f_y)$ are the Fourier transform of $g(x,y)$, $a(x,y)$, $c(x,y)$ and $c^{*}(x,y)$, respectively. Assuming that the terms, $a(x,y)$, $b(x,y)$, and $\phi(x,y)$, are slow varying functions compared with the spatial carrier-frequency f_{0x} and f_{0y}. The terms $A(f_x,f_y)$, $C(f_x-f_{0x},f_y-f_{0y})$ and $C^{*}(f_x+f_{0x},f_y+f_{0y})$ in the right hand of Eq. (4) are separate and do not overlap in the frequency domain. Hence the spectrum component $C(f_x-f_{0x},f_y-f_{0y})$ can be isolated with a suitable spectral filter. Then the component $C(f_x-f_{0x},f_y-f_{0y})$

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is shifted to the origin position with a distance of f_{0x} and f_{0y} along x and y direction, respectively, in the frequency domain, and we have $C(f_x,f_y)$. Taking the inverse Fourier transform of $C(f_x,f_y)$, $c(x,y)$ will be obtained then the measured phase component $\phi(x,y)$ is given by

$$\phi(x,y) = \tan^{-1}\frac{Re\,[c(x,y)]}{Im\,[c(x,y)]} \tag{5}$$

where $\tan^{-1}[\cdot]$ denotes the arctangent operator, $\mathrm{Im}[\cdot]$ and $\mathrm{Re}[\cdot]$ denote the imaginary and real parts of $c(x,y)$, respectively.

However, the fringe patterns are usually recorded by 2D solid-state image sensors, such as CCD camera, in practical applications. As discussed in Ref. [4], the digitization of the interferogram data can seriously distort the retrieved phase $\phi(x,y)$, because the translation is constrained to integer values of the spatial frequency which results in the digitization of sampling frequencies in the frequency domain. The carrier-frequency f_{0x} and f_{0y} usually are not an integer but instead of a small fraction of the frequency interval. In this case, a considerable departure will appear between the discrete carrier-frequency, which is determined by the peak coordinate of the side-lobe, and the real carrier-frequency f_{0x} and f_{0y}. Therefore, a carrier-removal error will be introduced in the traditional spectrum-shifting method as described in Reference [1] and it will result in a seriously tilt error in the retrieved phase $\phi(x,y)$ determined in Eq. (5).

In the past two decades, several solutions [17–23] are developed to suppress the carrier-removal error problem. Bone et al. [5] constructs a carrier phase plane from an information-free region in the interferogram by the least-squares fit method, and the measured phase is obtained by subtracting the carrier phase in the spatial domain. However, the accuracy of the retrieved phase is determined by the information-free region in the interferogram and the requirement of an information-free region in the interferogram cannot always be fulfilled [17]. Similarly, Gu and Chen [19] presents a bilinear surface to describe the carrier component. Another carrier-removal approach described in Ref. [20] directly removes the carrier phase in the spatial domain by subtracting a reference carrier phase calculated from an additional reference interferogram. For the same case, Nicola and Ferraro [21] remove the carrier phase in the frequency domain with FT method, and it also requires an additional pure carrier-frequency interferogram. However, recording two individually interferograms, a deformation interferogram and a reference interferogram, are required and negating the advantage of single-shot measurement in the FT method. Moreover, Ge [22] adjusts the carrier frequency values equal to an integral multiple of the sampling frequency by adjusting the inclination angle of the reference mirror with a piezoelectric actuator. However, it makes the setup complicated and consumes more time. Recently, Fan et al. [23] report a spectrum centroid method for suppressing carrier-removal error in the FT method for carrier interferogram analysis, for simplicity, it is shorted as "SC" method. The carrier-frequency values of interferogram are estimated from the spectrum centroid of component $C(f_x-f_{0x},f_y-f_{0y})$, and the carrier phase is removed by shifting the $C(f_x-f_{0x},f_y-f_{0y})$ to the origin position in the frequency domain by multiplying the original interferogram with a constructed pure carrier phase wave in the spatial domain. However, its accuracy is limited by the spectrum distribution and is unstable when chooses different window sizes of spectral filter.

In this paper, we present an effective, straightforward and stable carrier-removal technique for carrier interferogram analysis based on zero padding method. The carrier-frequency values within a small fraction of an integral (or pixel) are estimated from the extrapolation fringe with FFT approach. Then the carrier phase is removed by subtracting a pure carrier-frequency phase constructed by the estimated carrier-frequency f_{0x} and f_{0y} in the spatial domain. For brevity and distinction, we refer to the proposed method as "ZP" method. The principle of the proposed method is described in Section 2. Numerical simulations and experiments are carried out to demonstrate performance of the proposed ZP method in Sections 3 and 4, respectively. We conclude all the paper in Section 5.

2. Theory analysis

2.1. Discrete Fourier transform for fringe pattern analysis

The intensity of the fringe pattern is usually recorded by a solid-state image sensor such as CCD camera in practical applications thus the fringe pattern described in Eq. (1) usually can be further expressed as discrete form

$$g(m,n) = a(m,n) + b(m,n)\cos\left[2\pi\left(\frac{u_0}{M}m + \frac{v_0}{N}n\right) + \phi(m,n)\right] \tag{6}$$

where m, n are integer; M, N are the sampling points on the x, y directions, respectively; the sampling intervals T_x and T_y both equal to 1; u_0 and v_0 are integer and the values of them are closed to the true carrier-frequency f_{0x} and f_{0y}, respectively. The discrete Fourier transform (DFT) of Eq. (6) is given by [24]

$$G\left(\frac{u}{M},\frac{v}{N}\right) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} g(m,n)\exp\left[-j2\pi\left(\frac{u}{M}m + \frac{v}{N}n\right)\right] \tag{7}$$

where $G(u/M, v/N)$ is the spectral distribution of Eq. (6); u and v are integer of the sampling frequency interval; and $j=(-1)^{1/2}$. Similar to the analysis procedure in Section 1, the spectrum component $C(u/M-u_0/M, v/N-v_0/N)$ in Eq. (7) is extracted with a band filter in frequency domain and its inverse discrete Fourier transform (IDFT) is given by

$$\begin{aligned} c(m,n) &= \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} C\left(\frac{u-u_0}{M},\frac{v-v_0}{N}\right) e\left[j2\pi\left(\frac{u}{M}m + \frac{v}{N}n\right)\right] \\ &= \frac{1}{2}b(m,n)\exp\left[j2\pi\left(\frac{u_0}{M}m + \frac{v_0}{N}n\right) + \phi(m,n)\right] \end{aligned} \tag{8}$$

According to Eq. (8), the measured phase can be determined by

$$\phi(m,n) = \tan^{-1}\left\{\frac{Im\,[c(m,n)]}{Re\,[c(m,n)]}\right\} - 2\pi\left(\frac{u_0}{M}m + \frac{v_0}{N}n\right) \tag{9}$$

According to analysis in Section 1, the carrier-frequency f_{0x}, f_{0y} of carrier interferogram expressed in formula (2) usually are not equal to an integer multiple of the Niquest basic frequency $1/M$, $1/N$, thus they can be expressed as follow

$$\begin{cases} f_{0x} = \dfrac{u_0+\delta_x}{M}, & -0.5 < \delta_x < 0.5 \\ f_{0y} = \dfrac{v_0+\delta_y}{N}, & -0.5 < \delta_y < 0.5 \end{cases} \tag{10}$$

The deviation between actual carrier-frequency f_{0x}, f_{0y} and the discrete ones u_0, v_0, which caused by the discreteness of the sampling points in the spatial domain of fringe pattern, can be expressed as

$$\begin{cases} \delta_{fx} = f_{0x} - \dfrac{u_0}{M} = \dfrac{\delta_x}{M} \\ \delta_{fy} = f_{0x} - \dfrac{v_0}{N} = \dfrac{\delta_y}{N} \end{cases} \tag{11}$$