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Effects on squeezing and sub-poissonian of light in fourth harmonic generation up to first-order Hamiltonian interaction

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ABSTRACT

The effects on squeezing and sub-poissonian of light in fourth harmonic generation (FHG) are investigated based on the fully quantum mechanically up to the first order Hamiltonian interaction in gt, where g is the coupling constant between the modes per second and t is the interaction time between the waves during the process in a nonlinear medium. FHG is a process in which an incident laser beam of the fundamental frequency ω interacts with a nonlinear medium to produce the harmonic frequency at 4ω. The coupled Heisenberg equations of motion involving real and imaginary parts of the quadrature operators are established. The occurrence of amplitude squeezing effects in both the quadratures of the radiation field in the fundamental mode is investigated and found to be dependent on the selective phase values of the field amplitude. The photon statistics of the pump mode in this process have also been investigated and found to be sub-poissonian in nature. It is found that there is no possibility to produce squeezed light in the harmonic mode up to first-order interaction in gt. Further, we have found the case up to second-order Hamiltonian interaction in gt that the normal squeezing in the harmonic mode is directly depends upon the fourth-order squeezing of the initial pump field. This gives a method of converting higher-order (fourth-order) squeezing into normal squeezing in the harmonic mode and vice versa.

1. Introduction

Squeezing and sub-poissonian photon statistics of the electromagnetic field [1], which is a purely quantum mechanical phenomenon [2] has attracted considerable attention owing to its low-noise property [3,4] with applications in high quality telecommunication [5], quantum cryptography [6], etc. Squeezing has been either experimentally observed or theoretically predicted in a variety of nonlinear optical processes, such as harmonic generation [7,8], multi-wave mixing processes [9–11], Raman [12,13], hyper-Raman [14], etc. Hong and Mandel [15] and Hillery [16] have introduced the notion of amplitude squeezing of the quantized electromagnetic field in various nonlinear optical processes. Squeezing and photon statistical effect of the field amplitude in harmonic generation has also been reported by Mandel [17]. Higher-order sub-poissonian photon statistics of light have also been studied by Kim and Yoon [18]. Recently, Prakash and Mishra [19] have reported the higher-order sub-poissonian photon statistics and their use in detection higher-order squeezing. The nonclassical phenomena squeezing of radiation and photon statistics effects are expected to manifest itself in optical processes in which the nonlinear response of the system to the radiation field plays a great role. It also represents a new type of quantum state of the electromagnetic field and it has always been of interest to the research community in the fields of quantum optics, nonlinear optics, atomic physics, molecular physics and biological physics; hence their study can be expected to lead to new fundamental insights.

The aim of this paper is to study further properties of amplitude squeezing and sub-poissonian effects of the electromagnetic field in the fundamental mode including harmonic mode in fourth-harmonic generation process under short-time approximation based on a fully quantum mechanical approach up to first order in gt. The paper is organized as follows: Section 2 gives the definitions of squeezing and sub-poissonian states of light. We establish the analytic expression of selective phase angle dependent amplitude squeezing and sub-poissonian light in the fundamental mode up to first-order in gt in Section 3. The photon statistics of the pump mode in this process have also been incorporated in this section and found to be sub-poissonian in nature. In Section 4, we study

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the occurrence of amplitude squeezing effects in both the quadrature of the radiation field in the harmonic mode and found to be dependent on fourth-order squeezing of the fundamental mode. Finally, we conclude the paper in Section 5.

2. Definitions of amplitude squeezing and sub-poissonian states of light

Squeezed states of light are characterized by reduced quantum fluctuations in one quadrature of the field at the expense of the increased fluctuations in the other quadrature. It is possible to characterize the amplitude by its real and imaginary parts as

$$X_1 = \frac{1}{2}(A + A^\dagger) \quad \text{and} \quad X_2 = \frac{1}{2}(A + A^\dagger) \tag{1}$$

where A and $A^\dagger$ are the slowly varying operators useful in discussing squeezing effects. For a single mode of the electromagnetic field with frequency ω and creation (annihilation) operators $a^\dagger$ (a), they are given by

$$A = a\exp(i\omega t), \qquad A^\dagger = a^\dagger \exp(-i\omega t) \tag{2}$$

The operators defined by Eq. (1) do not commute and obey the commutation relation

$$[X_1, X_2] = \frac{1}{2} \tag{3}$$

and, as a result, satisfy the uncertainty relation ($\hbar = 1$)

$$\Delta X_1\, \Delta X_2 \geq \frac{1}{4} \tag{4}$$

where ΔX_1 and ΔX_2 are the uncertainties in the quadrature operators X_1 and X_2, respectively. A quantum state is squeezed in the X_1 direction if $\Delta X_1 < 1/2$ and is squeezed in the X_2 direction if $\Delta X_2 < 1/2$.

In order to define higher-order squeezing, i.e. amplitude squared squeezing [14] is defined in terms of operators Y_1 and Y_2 as

$$Y_1 = \frac{1}{2}(A^2 + A^{\dagger 2}) \quad \text{and} \quad Y_1 = \frac{1}{2i}(A^2 - A^{\dagger 2}) \tag{5}$$

The commutation relation of Eq. (5) leads to the uncertainty relation

$$\Delta Y_1\, \Delta Y_2 \geq \left\langle \left(N_A + \frac{1}{2} \right) \right\rangle \tag{6}$$

where ΔY_1 and ΔY_2 are the uncertainties in the quadrature operators Y_1 and Y_2, respectively. A quantum state is squeezed in the Y_1 direction if $(\Delta Y_1)^2 < \langle (N_A + (1/2)) \rangle$ and is squeezed in the Y_2 direction if $(\Delta Y_2)^2 < \langle (N_A + (1/2)) \rangle$.

Similarly, amplitude-cubed squeezing [14,17] is defined by the operators

$$Z_1 = \frac{1}{2}(A^3 + A^{\dagger 3}) \quad \text{and} \quad Z_2 = \frac{1}{2i}(A^3 - A^{\dagger 3}) \tag{7}$$

and the commutation relation of (7) follows the uncertainty relation as

$$\Delta Z_1\, \Delta Z_2 \geq \frac{1}{4} \langle (9N_A^2 + 9N_A + 6) \rangle \tag{8}$$

Hence the amplitude-cubed squeezing is said to exist if

$$(\Delta Z_1)^2 \quad \text{or} \quad (\Delta Z_2)^2 < \frac{1}{4} \langle (9N_A^2 + 9N_A + 6) \rangle \tag{9}$$

Now, it is possible to characterize the fourth-order amplitude by its real and imaginary parts as

$$F_1 = \frac{1}{2}(A^4 + A^{\dagger 4}) \quad \text{and} \quad F_2 = \frac{1}{2i}(A^4 - A^{\dagger 4}) \tag{10}$$

Then the fourth-order amplitude squeezing [14] state is said to exist if

$$(\Delta F_1)^2 \quad \text{and} \quad (\Delta F_2)^2 < \frac{1}{4} \langle 16N_A^3 + 24N_A^2 + 56N_A + 24 \rangle \tag{11}$$

The quantum effect of sub-poissonian photon statistics is the reduction of quantum fluctuations in photon number is reflected by an increase of fluctuations of phase of the field. Hence the photon number uncertainty is

$$\langle (\Delta N)^2 \rangle < \langle N \rangle \tag{12}$$

3. Effects on squeezing and sub-poissonian of light in the fundamental mode

FHG is a process in which an incident laser beam of the fundamental frequency ω_1 interacts with a nonlinear medium to produce the harmonic frequency at $\omega_2 = 4\omega_1$. This model is chosen to make the model realistic.

In this model (Fig. 1), the interaction is looked upon as a process involving absorption of four pump photons of frequency ω_1 each and system going from $|1\rangle$ to $|2\rangle$ and emission of one photon of frequency ω_2 and the atomic system finally coming back to the initial state $|1\rangle$.

The Hamiltonian for this process can be written as ($\hbar = 1$)

$$H = \omega_1 a^\dagger a + \omega_1 b^\dagger b + g(a^4 b^\dagger + a^{\dagger 4} b) \tag{13}$$

where $a^\dagger(a)$ and $b^\dagger(b)$ are the creation (annihilation) operators of the pump field (A-mode) and harmonic field (B-mode) respectively and g is the coupling constant in the interaction Hamiltonian, which is assumed real, describes the coupling between the two modes of the order of 10^2 to 10^4 per second and also proportional to the nonlinear susceptibility of the medium as well as the complex amplitude of the pump field [20,21]. In this model A and B are slowly varying operators useful to discuss squeezing defined as, $A = a\exp(i\omega_1 t)$ and $B = b\exp(i\omega_2 t)$ with the relation $\omega_2 = 4\omega_1$.

Using the interaction Hamiltonian of Eq. (13) in the coupled Heisenberg equation of motion

$$\dot{A} = \frac{\partial A}{\partial t} + i[H, A] \quad (\hbar = 1) \tag{14}$$

we obtain

$$\dot{A} = -4igA^{\dagger 3}B \tag{15}$$

Similarly, we have

$$\dot{B} = -igA^4 \tag{16}$$

The interaction time is taken to be short, of the order of 10^{-10} s and a nanosecond or picosecond pulse laser can be used as the pump field. For real physical situation in the short-time scale $gt \ll 1$ ($gt \sim 10^{-6}$) and the number of photons are very large ($|\alpha|^2 \gg 1$), it is

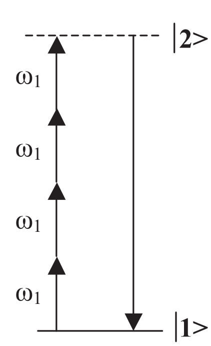

Fig. 1. Fourth harmonic generation model.