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Consensus of leader-followers system of multi-missile with time-delays and switching topologies

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ABSTRACT

The consensus problem in directed networks with arbitrary finite time-varying communication delays under both fixed topology and switching topologies is investigated in this article. The dynamics of each missile in this leader-followers system is with linear form. Feedback linearization is used here to attain linear guidance law for each missile, which is the base law for cooperative. Based on graph theory, the consensus problem can be converted to the stability of corresponding error system. Then Lyapunov function method is used to analyze the stability of the error system. Consensus of networks with time-delays under switching topologies is proved using common Lyapunov function method. Simulations indicate the excellent performances of the algorithms in terms of accuracy and efficiency.

1. Introduction

In recent years salvo attack of multiple missiles has been an effective measure to improve the performance [1,2]. Although each individual missile has limited processing power, the interconnected system as a whole can perform complex tasks in a coordinated fashion. Comparing with conventional control systems, multi-missile systems have many advantages such as reducing cost, improving system efficiency, flexibility, reliability, and providing new capability [3].

The systems in networked communications are considered as agents forming a network and information is exchanged between these agents through the network. Each agent only gets information from a set of neighbors.

There are two types of consensus protocols with time-delays. One is symmetric structure, which affects both the information of each agent and its neighbors [4–7]. The other one is the delays only affect the information of neighbors of one agent, which called as asymmetric structure [8–11]. Latest research results of consensus problems with time-delays can be found in [16,18–20].

The consensus of a group of agents with a leader, where the leader's dynamic is independent of the others, is now researched to realize the convergence of the whole network to a specific trajectory. Such a problem is commonly called leader-followers consensus problem. Reference [17] study two types of leaders, knowledge leader and power leader, each has own different function. The leader-followers consensus problem of a group of second-order dynamic agents with multiple time-varying delays as well as fixed and switching topologies is considered in [12]. A multi-agent system with a varying-velocity leader and time-varying delays is studied in [13]. Most of the leader-followers system is double integrator system. Reference [14] analyze a class of neutrally stable system, carrying out a state transformation on the model of the agents to proceed with the design.

This thesis first introduces a new kind of guidance law based on feedback linearization, which turns complex guidance problem to a very simple form. Based on the closed guidance system, an important problem for multi-missile system is to design simple control law for each agent, using local information from its neighbors or the leader if there exists one. In this paper, we consider the leader-followers consensus problem, where the dynamics of each agent is the linear guidance law we design in the context. Achieving consensus under fixed interaction topology with no time-delays is relatively easy. However, communication time-delays in networks maybe have a serious impact on finishing coordinated attacking task. We study consensus problems with fixed time-delays by transforming the stability problem to the problem of solving linear matrix inequalities. The network topology may change because of the limitation of communication distance. The consensus problem under switching topologies is also considered in this paper.

2. Problem formulation and preliminaries

We use graph theory to describe the information exchange between follower-follower and the follower–leader. The

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interaction topology of information exchange between followers is described by graph $G=(V, E, A)$, where $V=\{v_i : i=1, 2, \ldots, N\}$ is the set of N followers, and $E \subset V \times V$ is the set of edges of the graph. An edge of G is denoted by (i,j), representing that agents i and j can exchange information between them. Two nodes i and j are neighbors to each other if $(i, j) \in E$. The set of neighbors of node i is denoted by $N_i = \{j \in V : (j, i) \in E, j \neq i\}$. The leader is represented by vertex 0 and information is exchanged between the leader and the followers that are in the neighbors of the leader. Then, we have a graph $\bar{G}$, which consists of graph G, vertex 0 and edges between the leader 0 and its neighbors.

The adjacency matrix A of a graph G on vertex $\{1, \ldots N\}$ is a $N \times N$ matrix, whose (i, j)th entry is 1 if (i,j) is an edge of G and 0 if it is not. The degree matrix D of G is a diagonal matrix whose ith diagonal element is $N_i = \sum_{j=1}^{N} a_{ij}$. The Laplacian matrix of G is defined to be $L=[l_{ij}] \in R^{N \times N}$ such that $L=-A+D$.

Consider a multi-missile system consisting of N followers and a leader. The dynamics of each follower is:

$$x_i = Ax_i + Bu_i \tag{1}$$

where $\boldsymbol{x}_i \in R^n$ is the state of the ith missile, and $u_i \in R^m$ is the coordinate input of the ith missile which can only use local information from its neighbor followers or the leader. The matrix B is of full column rank. The leader, labeled as $i=0$, has linear dynamics as

$$\dot{x}_0 = Ax_0 \tag{2}$$

where $x_0 \in R^n$ is the state of the leader. Obviously, the leader's dynamics is independent of others.

In this paper, we consider the problem of designing $u_i, i=1,2,\ldots,N$ to make all N agents to follow the leader. We give the following definition to explain this.

Definition 1. The leader-followers consensus of system (1)–(2) is said to be achieved if, for each agent $i \in \{1, \ldots, N\}$, there is a local state feedback controller u_i of $\{x_j : j \in N_i\}$ such that the closed-loop system satisfies:

$$\lim_{t \to \infty} \left\| x_i(t) - x_0(t) \right\| = 0, \, i=1, \ldots N \tag{3}$$

for any initial condition $x_i(0)$, $i=0, 1, \ldots N$.

The following assumption is used throughout the paper.

Assumption 1. The pair (A,B) is stabilizable.

The following lemmas play an important role in the proof of the main results.

Lemma 1. *(Schur complement, Boyd et al. [15]). Let M, P, Q be given matrices, and $Q>0$. Then*

$$\begin{bmatrix} P & M \\ M^T & -Q \end{bmatrix} < 0 \Leftrightarrow P + MQ^{-1}M^T < 0. \tag{4}$$

3. Guidance law

The principle of cooperative guidance in heterogeneous multi-missile network with local communication is shown in Fig. 1.

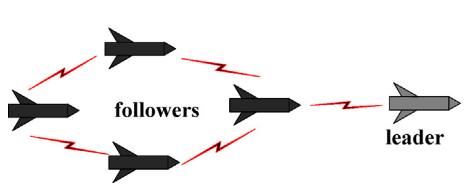

Fig. 1. Principle of cooperative guidance in heterogeneous multi-missile network with local communication.

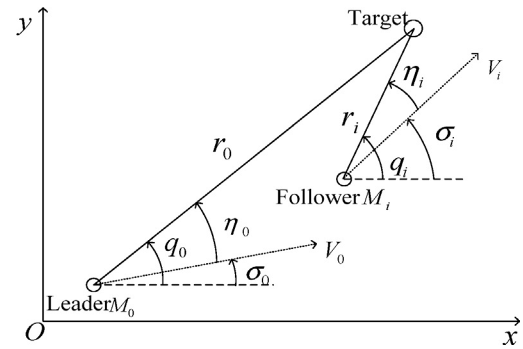

Fig. 2. Relative motion of leader-followers and target.

Adjacent communication in line-of-sight is used between two missiles. The leader whose dynamics is independent flights along the given guidance law.

The guidance problem can be formulated as a tracking problem for a time-varying nonlinear system. Fig. 2 is the relative motion of leader-followers and target.

We can get the guidance relationship of the leader and the target.

$$\begin{cases} \dot{r}_0 = -V_0 \cos \eta_0 \\ r_0 \dot{q}_0 = V_0 \sin \eta_0 \\ q_0 = \sigma_0 + \eta_0 \\ \dot{\sigma}_0 = \dfrac{a_0}{V_0} \end{cases} \tag{5}$$

where V_0 and a_0 are, respectively, the speed and acceleration of the leader. Other symbols' meaning can be seen from Fig. 2. Choose $\boldsymbol{x}_0 = [r_0 \, \eta_0]^T$ and the equation above is simplified as:

$$\begin{cases} \dot{r}_0 = -V_0 \cos \eta_0 \\ \dot{\eta}_0 = \dfrac{V_0 \sin \eta_0}{r_0} - \dfrac{a_0}{V_0} \end{cases} \tag{6}$$

Assume there are n followers in this system, each of which can be depicted as:

$$\begin{cases} \dot{r}_i = -V_i \cos \eta_i \\ \dot{\eta}_i = \dfrac{V_i \sin \eta_i}{r_i} - \dfrac{a_i}{V_i} \end{cases} \quad i = 1, 2, \ldots, n \tag{7}$$

Based on the method of feedback linearization for nonlinear system, define the following transformation:

$$\begin{cases} z_{i1} = r_i \\ z_{i2} = -V_i \cos \eta_i \end{cases} \tag{8}$$

Then the Eq. (7) can be transformed into:

$$\begin{cases} \dot{z}_{i1} = \dot{r}_i \\ \dot{z}_{i2} = \dfrac{V_i^2 \sin^2 \eta_i}{r_i} - u_i \sin \eta_i \end{cases} \tag{9}$$

Define $\boldsymbol{z}_i = [z_{i1} \; z_{i2}]^T$ and we can get:

$$\dot{\boldsymbol{z}}_i = F\boldsymbol{z}_i + W\gamma_i[u_i - \alpha_i] \tag{10}$$

where

$$\boldsymbol{F} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \boldsymbol{W} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$