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The influence of non-Kolmogorov turbulence on the entanglement of spatial two-qubit states in a slant channel

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ABSTRACT

The effects of atmospheric turbulence on the entanglement of spatial two-qubit states that are prepared using the signal and idler photons produced by parametric down-conversion are studied. Utilizing the non-Kolmogorov model for atmospheric turbulence and Rytov approximation method, we quantify the effects of atmospheric turbulence on the entanglement of the two-qubit state in terms of Wootters's concurrence. Our results show that the effects of the zenith angle of communication channel and the outer scale of turbulence on the concurrence of a spatial two-qubit state can be ignored and the smaller inner scale of turbulence, the smaller refractive-index power α, the shorter wavelength of beams and the longer propagation distance will lead to the larger fluctuations of the concurrence of a spatial two-qubit state.

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1. Introduction

Quantum communication utilizing entangled two-qubit states has many applications such as quantum teleportation [1], quantum cryptography [2,3] and quantum superdense coding [4]. Several publications have dealt with the influence of atmospheric turbulence on quantum entanglement [5–7]. In these studies, atmospheric turbulence has been taken to be the Kolmogorov model and the optical effects of the atmosphere at any moment have been described as a random phase operation $e^{i\varphi(\boldsymbol{r})}$, which are generally limited to short distance and horizontal propagation paths [8]. However, many new analytic results based on the non-Kolmogorov turbulence spectrum model and the Rytov approximation method have been published in recent years [9–16] because there are important application areas like certain ground/space links involving laser satellite communications for which weak fluctuation theory may be applied. In this paper, we investigate the influence of non-Kolmogorov turbulence on the entanglement of spatial two-qubit state in a slant channel based on the Rytov approximation method. In Section 2, the effects of the outer scale and the inner scale of turbulence and the zenith angle of communication channel on the entanglement of a two-qubit state are investigated in detail. Numerical results are given in Section 3. Conclusions are presented in Section 4.

2. The influence of non-Kolmogorov turbulence on the entanglement of spatial two-qubit states

Fig. 1 depicts a generic scheme for preparing spatial two-qubit states using the entangled photons produced by parametric down-conversion (PDC). Jha and Boyd have analyzed it in detail [17]. The signal and idler photons produced by PDC go through a pair of double-holes located at plane z. They are detected in coincidence by detectors D_s and D_i located at positions $\boldsymbol{r}_s$ and $\boldsymbol{r}_i$, respectively. The spacing between the two signal and the idler holes is taken to be much bigger than the two-photon correlation width $\sigma_s^{(2)}(z)$ so that the two-photon spectral densities for the pairs of transverse positions $(\boldsymbol{\rho}_{s1}, \boldsymbol{\rho}_{i2})$ and $(\boldsymbol{\rho}_{s2}, \boldsymbol{\rho}_{i1})$ are negligibly small.

With the above assumption, the density matrix ρ_{qubit} of the two-qubit state can be represented by a density matrix having only two non-zero diagonal elements. Therefore, the concurrence C_{qubit}, which is a well-established method defined by Wootters [18,19] for quantifying the degree of entanglement of a two-qubit state, can be shown to be [5]

$$C_{\text{qubit}} = 2\eta k_1 k_2 W^{(2)}(\boldsymbol{r}_{s1}, \boldsymbol{r}_{i1}, \boldsymbol{r}_{s2}, \boldsymbol{r}_{i2}). \tag{1}$$

where $\eta = 1/[k_1^2 S^{(2)}(\boldsymbol{r}_{s1}, \boldsymbol{r}_{i1}) + k_2^2 S^{(2)}(\boldsymbol{r}_{s2}, \boldsymbol{r}_{i2})]$ is a constant of proportionality, $W^{(2)}(\boldsymbol{r}_{s1}, \boldsymbol{r}_{i1}, \boldsymbol{r}_{s2}, \boldsymbol{r}_{i2})$ is the two-photon cross-spectral

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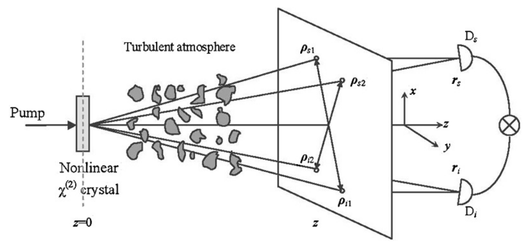

Fig. 1. Illustration of the notation relating to spatial two-qubit states propagation through atmospheric turbulence.

density function [17], $S^{(2)}(\boldsymbol{r}_{s1}, \boldsymbol{r}_{i1}) = W^{(2)}(\boldsymbol{r}_{s1}, \boldsymbol{r}_{i1}, \boldsymbol{r}_{s1}, \boldsymbol{r}_{i1})$, and the constant factors k_1, k_2 depend on the sizes of the apertures and the geometry of the arrangement.

In the turbulent atmosphere the two photon cross-spectral density function $W^{(2)}(\boldsymbol{r}_{s1}, \boldsymbol{r}_{i1}, \boldsymbol{r}_{s2}, \boldsymbol{r}_{i2})$ can be defined as

$$W^{(2)}(\boldsymbol{r}_{s1}, \boldsymbol{r}_{i1}, \boldsymbol{r}_{s2}, \boldsymbol{r}_{i2}) = \langle \mathrm{tr}\{\rho_{\mathrm{tp}} \hat{\hat{E}}_{s1}^{(-)}(\boldsymbol{r}_{s1}) \hat{\hat{E}}_{i1}^{(-)}(\boldsymbol{r}_{i1}) \hat{\hat{E}}_{i2}^{(+)}(\boldsymbol{r}_{i2}) \hat{\hat{E}}_{s2}^{(+)}(\boldsymbol{r}_{s2})\}\rangle. \tag{2}$$

Here $\hat{\hat{E}}_{s2}^{(+)}(\boldsymbol{r}_{s2})$ is the positive frequency part of the field at position $\boldsymbol{r}_{s2}$, etc. The symbol tr stands for the trace, and ρ_{tp} is the density matrix of the two-photon field produced by PDC. The ensemble average $\langle \cdots \rangle$ is to account for the statistical fluctuations introduced by the turbulent medium.

Using the Rytov approximation [20], the field $\hat{\hat{E}}_{s1}^{(+)}(\boldsymbol{r}_{s1})$ at position $\boldsymbol{r}_{s1} = (\boldsymbol{\rho}_{s1}, z)$ can be written as

$$\hat{\hat{E}}_{s1}^{(+)}(\boldsymbol{r}_{s1}) = \hat{E}_{s1}^{(+)}(\boldsymbol{r}_{s1}) \exp[\psi(\boldsymbol{r}_{s1})]. \tag{3}$$

Here $\hat{E}_{s1}^{(+)}(\boldsymbol{r}_{s1})$ represents a deterministic field, whereas the function $\psi(\boldsymbol{r}_{s1}) = \chi(\boldsymbol{r}_{s1}) + is(\boldsymbol{r}_{s1})$ describes the effects of the atmospheric turbulence on the propagation of a spherical wave, $\chi(\boldsymbol{r}_{s1})$ and $s(\boldsymbol{r}_{s1})$ are terms imposed by atmospheric turbulence and account for the stochastic log-amplitude and phase fluctuations, respectively. Then Eq. (2) becomes

$$W^{(2)}(\boldsymbol{r}_{s1}, \boldsymbol{r}_{i1}, \boldsymbol{r}_{s2}, \boldsymbol{r}_{i2}) = \mathrm{tr}\{\rho_{\mathrm{tp}} \hat{E}_{s1}^{(-)}(\boldsymbol{r}_{s1}) \hat{E}_{i1}^{(-)}(\boldsymbol{r}_{i1}) \hat{E}_{i2}^{(+)}(\boldsymbol{r}_{i2}) \hat{E}_{s2}^{(+)}(\boldsymbol{r}_{s2})\} \times \langle \exp[\psi^*(\boldsymbol{r}_{s1}) + \psi^*(\boldsymbol{r}_{i1}) + \psi(\boldsymbol{r}_{i2}) + \psi(\boldsymbol{r}_{s2})]\rangle. \tag{4}$$

For pump beams that are of fully coherent Gaussian Schell-model type, and considering the special case $k_1 S^{(2)}(\boldsymbol{r}_{s1}, \boldsymbol{r}_{i1}) = k_2 S^{(2)}(\boldsymbol{r}_{s2}, \boldsymbol{r}_{i2})$, the concurrence of the spatial two-qubit state can be written as [5]

$$C_{\mathrm{qubit}} = \langle \exp[\psi^*(\boldsymbol{r}_{s1}) + \psi^*(\boldsymbol{r}_{i1}) + \psi(\boldsymbol{r}_{i2}) + \psi(\boldsymbol{r}_{s2})]\rangle. \tag{5}$$

Under weak turbulence, for which the wave-structure function is dominated by phase, the concurrence of the spatial two-qubit state can be written as [21]

$$C_{\mathrm{qubit}} = \exp\{-\frac{1}{2}[D_\psi(\boldsymbol{r}_{s1} - \boldsymbol{r}_{s2}) + D_\psi(\boldsymbol{r}_{i1} - \boldsymbol{r}_{i2}) + D_\psi(\boldsymbol{r}_{s1} - \boldsymbol{r}_{i2}) + D_\psi(\boldsymbol{r}_{s2} - \boldsymbol{r}_{i1}) - D_\psi(\boldsymbol{r}_{s1} - \boldsymbol{r}_{i1}) - D_\psi(\boldsymbol{r}_{s2} - \boldsymbol{r}_{i2})]\},$$

where

$$D_\psi(\boldsymbol{r}_{s1} - \boldsymbol{r}_{s2}) = \frac{2|\boldsymbol{\rho}_{s1} - \boldsymbol{\rho}_{s2}|^{5/3}}{\rho_0^{5/3}},$$

here ρ_0 is the lateral coherence length of the spherical wave. Then

$$C_{\mathrm{qubit}} = \exp[-\frac{1}{\rho_0^{5/3}}(|\boldsymbol{\rho}_{s1} - \boldsymbol{\rho}_{s2}|^{5/3} + |\boldsymbol{\rho}_{i1} - \boldsymbol{\rho}_{i2}|^{5/3} + |\boldsymbol{\rho}_{s1} - \boldsymbol{\rho}_{i2}|^{5/3} + |\boldsymbol{\rho}_{s2} - \boldsymbol{\rho}_{i1}|^{5/3} - |\boldsymbol{\rho}_{s1} - \boldsymbol{\rho}_{i1}|^{5/3} - |\boldsymbol{\rho}_{s2} - \boldsymbol{\rho}_{i2}|^{5/3})].$$

For conceptual clarity, we can also assume that the two pairs of signal and idler apertures are placed at symmetric positions, that is, $\boldsymbol{\rho}_{s1} = -\boldsymbol{\rho}_{i1}$ and $\boldsymbol{\rho}_{s2} = -\boldsymbol{\rho}_{i2}$. Then the concurrence C_{qubit} of the spatial two-qubit state can be simplified as

$$C_{\mathrm{qubit}} = \exp[-\frac{1}{\rho_0^{5/3}}(2|\Delta\boldsymbol{\rho}|^{5/3} + 2|\Delta\boldsymbol{\rho}'|^{5/3} - |\Delta\boldsymbol{\rho} + \Delta\boldsymbol{\rho}'|^{5/3} - |\Delta\boldsymbol{\rho} - \Delta\boldsymbol{\rho}'|^{5/3})] = \exp\{-\frac{1}{\rho_0^{5/3}}[2d_1^{5/3} + 2d_2^{5/3} - (d_1^2 + d_2^2 + 2d_1 d_2 \cos\phi)^{5/6} - (d_1^2 + d_2^2 - 2d_1 d_2 \cos\phi)^{5/6}]\},$$

here $\Delta\boldsymbol{\rho} = \boldsymbol{\rho}_{s1} - \boldsymbol{\rho}_{s2}$, $\Delta\boldsymbol{\rho}' = \boldsymbol{\rho}_{s1} - \boldsymbol{\rho}_{i2}$, $d_1 = |\Delta\boldsymbol{\rho}|$ can be taken as a measure of the effective physical size of the two-qubit state, and $d_2 = |\Delta\boldsymbol{\rho}'|$ can be taken as the measure of the separation between the two signal and the two idler apertures, and ϕ is the included angle between $\Delta\boldsymbol{\rho}$ and $\Delta\boldsymbol{\rho}'$.

For the non-Kolmogorov channel, the spectrum of atmospheric turbulence is represented as [9]

$$\phi_n(\kappa) = A(\alpha)\tilde{C}_n^2 \frac{\exp[-\kappa^2/\kappa_m^2]}{(\kappa^2 + \kappa_0^2)^{\alpha/2}}, \quad 0 \le \kappa < \infty, \quad 3 < \alpha < 5, \tag{6}$$

here α is the spectrum power of the refractive-index fluctuations(abbreviation as the refractive-index power), $A(\alpha) = (1/4\pi^2)\Gamma(\alpha - 1)\cos(\alpha\pi/2)$, with $\Gamma(\alpha)$ being the Gamma function, $\kappa_0 = 2\pi/L_0$, L_0 being the outer scale of turbulence, $\kappa_m = c(\alpha)/l_0$, l_0 being the inner scale of turbulence, $c(\alpha) = [\Gamma(5 - \alpha/2)A(\alpha)(2/3)\pi]^{1/(\alpha-5)}$, $\tilde{C}_n^2$ is a generalized refractive-index structure parameter with unit $\mathrm{m}^{3-\alpha}$, which is altitude dependent [12] and is given by

$$\tilde{C}_n^2(z\cos\theta) = 0.033(k\cos\theta/h)^{(\alpha/2-11/6)} \frac{[0.00594(v/27)^2 \times (h \times 10^{-5})^{10} \exp(-h/1000) + 2.7 \times 10^{-16} \times \exp(-h/1500) + C_n^2(0)\exp(-h/100)]}{A(\alpha)}, \tag{7}$$

where $k = 2\pi/\lambda$, λ is a wavelength of light, $h = z\cos\theta$ is altitude, v is the rms wind speed, $C_n^2(0)$ is the structure parameter at the ground and θ is the zenith angle of communication channel. Commonly used values are $v = 21$ m/s and $C_n^2(0) = 1.7 \times 10^{-14}$ $\mathrm{m}^{-2/3}$ in Hufnagel–Valley model. By the non-Kolmogorov spectrum Eq. (6), the lateral coherence length of the spherical wave is given by

$$\rho_0 = \left(\frac{A(\alpha)\pi^2 k^2 z}{2(\alpha - 2)}\{\kappa_m^{2-\alpha}[2\kappa_0^2 - (2-\alpha)\kappa_m^2]\exp\left(\frac{\kappa_0^2}{\kappa_m^2}\right)\gamma \times \left(2 - \frac{\alpha}{2}, \frac{\kappa_0^2}{\kappa_m^2}\right) - 2\kappa_0^{4-\alpha}\} \int_0^1 \tilde{C}_n^2(z\cos\theta\xi)(1-\xi)^2 d\xi\right)^{-1/2}, \quad 3 < \alpha < 4, \tag{8}$$

where $\gamma(a, x)$ is a incomplete gamma function.

Therefore, the concurrence of the two-qubit states in non-Kolmogorov atmospheric turbulence can now be written as:

$$C_{\mathrm{qubit}} = \exp\left[-\left(\frac{A(\alpha)\pi^2 k^2 z}{2(\alpha - 2)}\{\kappa_m^{2-\alpha}[2\kappa_0^2 - (2-\alpha)\kappa_m^2]\exp\left(\frac{\kappa_0^2}{\kappa_m^2}\right) \times \gamma\left(2 - \frac{\alpha}{2}, \frac{\kappa_0^2}{\kappa_m^2}\right) - 2\kappa_0^{4-\alpha}\} \int_0^1 \tilde{C}_n^2(z\cos\theta\xi)(1-\xi)^2 d\xi\right)^{5/6} \times [2d_1^{5/3} + 2d_2^{5/3} - (d_1^2 + d_2^2 + 2d_1 d_2\cos\phi)^{5/6} - (d_1^2 + d_2^2 - 2d_1 d_2\cos\phi)^{5/6}]\right]. \tag{9}$$

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