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Dynamics control of geometric quantum discord for two coupling qubits in a squeezed vacuum reservoir

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ABSTRACT

We investigate the dynamics of geometric quantum discord of coupled qubits in a squeezed vacuum reservoir. The results show that there is distinct difference between the dynamics of geometric quantum discord and that of quantum entanglement near (or away from) the decoherence free subspace. We also find that the squeezed vacuum reservoir with high squeezed amplitude is more suitable for geometric quantum discord to survive. The robustness of geometric quantum discord is stronger than that of quantum entanglement.

1. Introduction

Entanglement is regarded as the key resource of quantum information processing. There are many interesting applications based on these entangled systems, such as dense coding, quantum teleportation and quantum cryptography. An important feature of entanglement is that it gives rise to correlations, which cannot be explained by any local realistic description of quantum mechanics. Therefore, quantum entanglement is a non-local quantum connection. However, decoherence is a ubiquitous phenomenon in quantum mechanics and any architecture of future quantum computers must consider it seriously. So, investigating and quantifying the amount of entanglement contained in entangled system interacting with open systems is very important in the context of quantum information [1–4].

It is well known that quantum entanglement is not the only kind of quantum correlations for quantum information processing. It has shown both theoretically and experimentally that some tasks can be speeded up over their classical counterparts using fully separable and highly mixed states. To generally quantify the quantum correlations contained in bipartite systems, Ollivier and Zurek have defined a measure known as quantum discord [5]. It is largely accepted that the total correlations contained in quantum states can be described by quantum mutual information which is the sum of the quantum discord and classical correlation given by Vedral [6]. In general, there are two kinds of quantum discord: measurement-based discord and distance-based discord. The original definition of discord given by Ollivier and Zurek is the measurement based discord. The distance-based discord is adopted in Refs. [7,8]. This discord is defined as the minimal distance of a quantum state and all states with zero discord. Comparing with the original definition of quantum discord, this definition helps us to get an analytical expression for arbitrary two-qubit systems. This kind of measure is also called the geometric measure of quantum discord (geometric quantum discord) since it is similar to the geometry measure of quantum entanglement [9].

Up to now, difference schemes have been derived to remove the effects produced by the environment, for example, quantum error correction, decoherence-free subspace (DFS), and dynamical decoupling and quantum Zeno effect [10]. Recently, decoherence free entanglement was investigated for two qubits interacting with a common squeezed vacuum bath. It claimed that for states belonging initially to the DFS plane, the phenomenon of entanglement sudden death never occurs [11]. Moreover, for states outsides the DFS plane, the dynamics of entanglement displays two different types of behavior, namely the phenomena of entanglement decay and entanglement sudden death. Based on the fundamental and practical values, a natural question emerges if the DFS could avoid the death of quantum discord.

In this paper, the Markovian decoherence of the geometric quantum discord is obtained by using the master equation when the squeezed reservoirs are considered. The time-dependent behavior of geometric quantum discord is analytically and numerically

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analyzed. The squeezed property of the geometric quantum discord is also investigated.

2. Model

The Markovian master equation, in the interaction picture, for two qubits interacting with a squeezed vacuum bath (at zero temperature) is given by [12]

$$\frac{\partial \rho_{AB}}{\partial t} = \frac{\gamma}{2}(2S\rho_{AB}S^{+} - S^{+}S\rho_{AB} - \rho_{AB}S^{+}S), \tag{1}$$

with

$$S = \sqrt{N+1}(\sigma_1^- + \sigma_2^-) - \sqrt{N}e^{i\theta}(\sigma_1^+ + \sigma_2^+), \tag{2}$$

where γ is the spontaneous emission rate and $N = \sin h^2 r$. r and θ are the squeeze parameters of the bath. $\sigma_j^{\pm}(j = A, B)$ is the Pauli operator which is used to describe the qubits which obey the commutation and anticommutation algebra relation of Pauli operators. For simplicity, we assume $\gamma = 1$.

The decoherence-free subspace (DFS) was found in Ref. [13] and is composed of all eigenstates of S with zero eigenvalue. The two orthogonal vectors in this DFS plane are

$$\left|\Phi_1\right\rangle = \frac{1}{\sqrt{N^2+M^2}}\left(N\left|11\right\rangle + Me^{i\theta}\left|00\right\rangle\right), \tag{3}$$

$$\left|\Phi_2\right\rangle = \frac{1}{\sqrt{2}}\left(\left|01\right\rangle - \left|10\right\rangle\right). \tag{4}$$

$M = \sqrt{N(N+1)}$. One can also define the states $\left|\Phi_3\right\rangle$ and $\left|\Phi_4\right\rangle$ orthogonal to the $\left\{\left|\Phi_1\right\rangle, \left|\Phi_2\right\rangle\right\}$ plane:

$$\left|\Phi_3\right\rangle = \frac{1}{\sqrt{2}}\left(\left|01\right\rangle + \left|10\right\rangle\right), \tag{5}$$

$$\left|\Phi_1\right\rangle = \frac{1}{\sqrt{N^2+M^2}}\left(M\left|11\right\rangle - Ne^{i\theta}\left|00\right\rangle\right). \tag{6}$$

Based on the basis $\left\{\left|\Phi_1\right\rangle, \left|\Phi_2\right\rangle, \left|\Phi_3\right\rangle, \left|\Phi_4\right\rangle\right\}$, the solutions of the quantum master equation can be found, and they depend on the initial values and squeeze parameters of the bath. In order to study the geometric quantum discord of two qubits in the common squeezed bath, we consider the initial states of the form

$$\left|\psi_1\right\rangle = \varepsilon\left|\Phi_1\right\rangle + \sqrt{1-\varepsilon^2}\left|\Phi_4\right\rangle, \tag{7}$$

$$\left|\psi_2\right\rangle = \varepsilon\left|\Phi_2\right\rangle + \sqrt{1-\varepsilon^2}\left|\Phi_3\right\rangle, \tag{8}$$

where ε is variable amplitude of one of the states belonging to the DFS. We would like to study the effect of varying ε on the quantum discord.

3. Quantum entanglement and geometric quantum discord for two qubits

In general, the density matrices for solutions of Eq. (1) are written in the standard basis of the form:

$$\rho_{AB}(t) = \begin{pmatrix} \rho_{11}(t) & 0 & 0 & \rho_{14}(t) \\ 0 & \rho_{22}(t) & \rho_{23}(t) & 0 \\ 0 & \rho_{32}(t) & \rho_{33}(t) & 0 \\ \rho_{41}(t) & 0 & 0 & \rho_{44}(t) \end{pmatrix}. \tag{9}$$

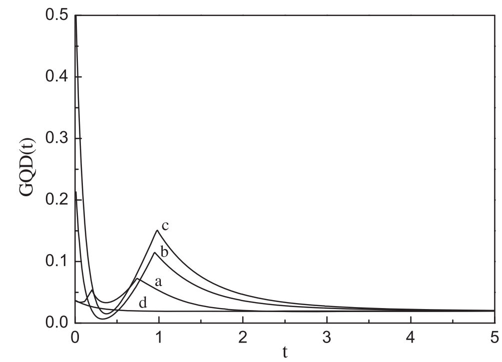

Fig. 1. Time evolution of geometric quantum discord $GQD(t)$ for initial state $\left|\psi_1(t)\right\rangle$ and $r = 0.1$: (a) $\varepsilon = 0.1$; (b) $\varepsilon = 0.3$; (c) $\varepsilon = 0.5$; (d) $\varepsilon = 1$.

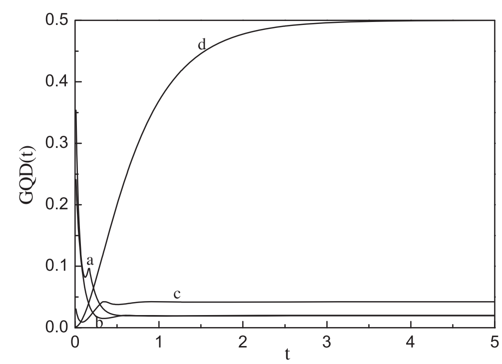

Fig. 2. Time evolution of geometric quantum discord $GQD(t)$ for initial state $\left|\psi_2(t)\right\rangle$ and $r = 0.1$: (a) $\varepsilon = 0.1$; (b) $\varepsilon = 0.3$; (c) $\varepsilon = 0.5$; (d) $\varepsilon = 1$.

The concept of geometric quantum discord is discussed in Ref. [14] to quantify the nonclassical correlations. For an arbitrary bipartite state $\rho_{AB}(t)$, geometric quantum discord (GQD) is defined as

$$D_A^g(\rho_{AB}) = \min_{\chi\in\Omega_0}\left\|\rho_{AB} - \chi\right\|^2 \tag{10}$$

where the minimum is bigger than all possible classical states χ of the form $p_1\left|\psi_1\right\rangle\left\langle\psi_1\right| \otimes \rho_A + p_2\left|\psi_2\right\rangle\left\langle\psi_2\right| \otimes \rho_B$ with $p_1 + p_2 = 1$. $\left|\psi_1\right\rangle$ and $\left|\psi_2\right\rangle$ are two orthonormal bases of subsystem A. ρ_A and ρ_B are two density matrices of subsystem $A(B)$. Here, $\left\|\rho - \chi\right\|^2 = Tr(\rho - \chi)^2$ is the square norm of Hilbert–Schimidt space.

In the following, we are going to discuss how the squeezed amplitude parameter affects geometric quantum discord of coupled qubits while the quantum system is initially in $\left|\psi_1\right\rangle$ and $\left|\psi_2\right\rangle$ states respectively. Figs. 1–6 show the most representative evolution curves. In both cases, we vary ε between 0 and 1 for a fixed squeezed amplitude parameter. We observe from Figs. 1 and 2 that the initial geometric quantum discord evolves to steady-state value asymptotically due to the dissipation effect. And the phenomenon of sudden death and revival never occurs. However, the correlation behavior of system would be different when we start from different initial states with fixed squeezed parameters. When the initial state is $\left|\psi_1\right\rangle$, the steady-state value of geometric quantum discord is independent of parameter ε. In other words, no matter getting close to or getting away from the DFS, geometric quantum discord has a fixed steady-state value. Inversely, when the initial state is $\left|\psi_2\right\rangle$, the steady-state value of geometric quantum discord depends on parameter ε and becomes bigger as getting closer to the DFS.

It is obvious that in squeezed vacuum reservoir, the dynamic behaviors of the geometric quantum discord are different from that