Optik 125 (2014) 1346–1353

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

# Observability analysis of navigation system using point-based visual and inertial sensors

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ARTICLE INFO

*Article history:*
Received 11 March 2013
Accepted 8 August 2013

*Keywords:*
Matrix Kalman filter
Lie derivatives
Observability of nonlinear systems
Point feature
Vision

ABSTRACT

A matrix Kalman filter (MKF) has been implemented for a navigation system using point-based visual and inertial sensors. The observability conditions have been proved by the observability rank criterion based on Lie derivatives. The conditions are: (a) at least one degree of rotational freedom is excited and (b) at least three observed points are not collinear where any two points are linearly independent. Experimental results have demonstrated that the proposed algorithm obtains the same accurate as the line-based algorithm.



## 1. Introduction

In most outdoor navigation applications, GPS/INS integrated system has been widely used. However, in urban environments and in doors navigation applications we cannot use GPS since its signals are unavailable. An alternative approach is using other sensors, such as cameras. Combining these two sensors to form a vision-aided inertial navigation system (V-INS) has recently become a popular topic of research [1]. Accurate 3-D orientation estimates of a rigid body by inertial/magnetic sensing were exploited [2]. Orientation estimates are based on the concept of vector matching, which requires, in principle, the measurements of constant reference vectors (e.g., gravity and the earth's magnetic field) [3].

In this paper, we adopt a matrix Kalman filter (MKF) in which the estimate of the state matrix is expressed in terms of the matrix parameters of the original plant [4]. Compared with the ordinary EKF, the MKF has the advantages of a compact matrix notation by expressing the estimated matrix in terms of the original plant parameters [5].

The major contribution of this paper is to elucidate under which conditions an indoor navigation system using point-based visual and inertial sensors is observable. The state matrix contains the body attitude matrix, the gyro bias vector, relative velocity vector, and the vector direction of the observed point. We have extended the current work for the observability analysis for an orientation system described in [4,6], to the navigation systems based on inertial/visual sensors. Compared with observability analysis of a MKF-based navigation system using visual/inertial/magnetic sensors [4], the difference in the paper are: (a) visual feature is point not line and (b) the number of sensors are reduced, navigation system only use visual and inertial sensors.

## 2. Sensor modeling

### 2.1. Inertial sensor

The kinematic equations are the same as in [4].

$$\dot{\xi}^n = -\mathbf{v}^n \tag{1}$$

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0030-4026/$ – see front matter 
http://dx.doi.org/10.1016/j.ijleo.2013.08.004

$$\dot{\mathbf{v}}^n = \mathbf{C}_b^n(\tilde{\mathbf{f}}^b - \mathbf{b}_a) + \mathbf{g}^n \tag{2}$$

$$\dot{\mathbf{C}}_n^b = -\left[\boldsymbol{\omega}_{nb}^b \times\right] \mathbf{C}_n^b,\ \boldsymbol{\omega}_{nb}^b = \tilde{\boldsymbol{\omega}}_{ib}^b - \mathbf{b}_g \tag{3}$$

$$\dot{\mathbf{b}}_g = \mathbf{n}_{\omega g} \tag{4}$$

where $\xi^n$ is an arbitrary point on the ground, $\mathbf{b}_a$ and $\mathbf{b}_g$ are the biases of the accelerometer and gyro measurements, respectively. $[\boldsymbol{\omega}_{nb}^b \times]$ is the skew symmetric matrix of $\boldsymbol{\omega}_{nb}^b$.

### 2.2. Visual sensor

Unit vector direction of the observed point is chosen as measurements.

$$\eta^c = -\frac{1}{r}\begin{bmatrix} u \\ v \\ f \end{bmatrix} \tag{5}$$

where $u$ and $v$ are image coordinates of $\xi^n$, $f$ is the focal length and $r$ is the norm of $u$, $v$ and $f$.

## 3. MKF algorithm

### 3.1. Process model

The evolving state is as follows:

$$\mathbf{X}_k = \begin{bmatrix} \mathbf{C}_{nk}^b & \mathbf{b}_{gk} & \mathbf{v}_k^n & \eta_{1k}^n & \eta_{2k}^n & \eta_{3k}^n \end{bmatrix}, \quad i = 1, 2, 3 \tag{6}$$

where $\eta_{ik}^n$ is the vector direction in the $n$-frame. The dynamic equation is

$$\dot{\eta}_i^n = -\frac{\mathbf{v}^n}{\left|\xi_i\right|} - \frac{\xi_i \cdot \mathbf{v}^n}{\left|\xi_i\right|^2}\eta_i^n, \quad i = 1, 2, 3 \tag{7}$$

According to the conclusion in [4], we obtain the dynamic model as:

$$\mathbf{X}_{k+1} = \sum_{r=1}^{8} \Theta_k^r \mathbf{X}_k \Lambda_k^r + \mathrm{B}_k \mathbf{U}_k \mathrm{E} + \mathbf{W}_k \tag{8}$$

where the dynamic matrices, $\Theta_k^r$, $\Lambda_k^r$, are defined as

$$\begin{aligned}
&\Theta_k^1 = \Phi_k && \Lambda_k^1 = \mathbf{E}^{11} + \mathbf{E}^{22} + \mathbf{E}^{33} \\
&\Theta_k^2 = -[\mathbf{c}_{1k}\times] && \Lambda_k^2 = \mathbf{E}^{41}\Delta t \\
&\Theta_k^3 = -[\mathbf{c}_{2k}\times] && \Lambda_k^3 = \mathbf{E}^{42}\Delta t \\
&\Theta_k^4 = -[\mathbf{c}_{3k}\times] && \Lambda_k^4 = \mathbf{E}^{43}\Delta t \\
&\Theta_k^5 = I_3 && \Lambda_k^5 = \mathbf{E}^{44} + \mathbf{E}^{55} - \mathbf{E}^{56}\frac{\Delta t}{\left|\xi_1\right|} - \mathbf{E}^{57}\frac{\Delta t}{\left|\xi_2\right|} - \mathbf{E}^{58}\frac{\Delta t}{\left|\xi_3\right|} \\
&\Theta_k^6 = \exp^{-(\xi_1 \cdot v^n/|\xi_1|)\Delta t}\mathbf{I}_3 && \Lambda_k^6 = \mathbf{E}^{66} \\
&\Theta_k^7 = \exp^{-(\xi_2 \cdot v^n/|\xi_2|)\Delta t}\mathbf{I}_3 && \Lambda_k^7 = \mathbf{E}^{77} \\
&\Theta_k^8 = \exp^{-(\xi_3 \cdot v^n/|\xi_3|)\Delta t}\mathbf{I}_3 && \Lambda_k^8 = \mathbf{E}^{88}
\end{aligned} \tag{9}$$

where $\mathbf{E}^{ij}$ denotes a $8 \times 8$ matrix with 1 at position $(ij)$ and 0 elsewhere;

$$\mathbf{B}_k = \begin{bmatrix} \mathbf{C}_{nk}^{bT} & \mathbf{I}_3 \end{bmatrix}, \mathbf{U}_k = \begin{bmatrix} \mathbf{f}_k^b \\ \mathbf{g}^n \end{bmatrix}, \quad \mathrm{E} = \begin{bmatrix} 0 & 0 & 0 & 0 & \Delta t & 0 & 0 & 0 \end{bmatrix} \tag{10}$$