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Self-focusing of Gaussian laser beam in collisionless plasma and its effect on stimulated Raman scattering process

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ABSTRACT

This paper presents an investigation of self-focusing of Gaussian laser beam in collisionless plasma and its effect on stimulated Raman scattering process. The pump beam interacts with a pre-excited electron plasma wave thereby generating a back-scattered wave. On account of Gaussian intensity distribution of laser beam, the time independent component of the ponderomotive force along a direction perpendicular to the beam propagation becomes finite, which modifies the background plasma density profile in a direction transverse to pump beam axis. This modification in density affects the incident laser beam, electron plasma wave and back-scattered beam. We have set up the non-linear differential equations for the beam width parameters of the main beam, electron plasma wave, back-scattered wave and SRS-reflectivity by taking full non-linear part of the dielectric constant of collisionless plasma with the help of moment theory approach. It is observed from the analysis that focusing of waves greatly enhances the SRS reflectivity.

1. Introduction

An efficient coupling of a high-power laser beam with plasma is a topic of current research interest in many areas such as laser induced fusion and particle acceleration [1–8]. In the laser plasma interaction process, various instabilities such as self-focusing, filamentation, stimulated Raman scattering (SRS), stimulated Brillouin scattering (SBS), two plasmon decay, etc. come in to picture [9–14]. Due to these instabilities, the energy of the high-power laser beam is not efficiently coupled with plasma. Therefore, these nonlinear phenomena are being studied theoretically and experimentally. In SRS process, the incident laser beam decays into a scattered wave and an electron plasma wave. The large amplitude electron plasma wave produces super thermal electrons that penetrate and pre-heat the target core and the scattered wave represents a substantial amount of wasted energy, i.e., the energy that would otherwise get coupled to the target. Therefore, to know about the amount of useful and dissipated energy in laser plasma interaction, SRS reflectivity becomes a very important parameter for the achievement of laser induced fusion.

Many theoretical investigations have been carried out on self-focusing and scattering of laser beam separately by ignoring the interplay among them. The evolution of these instabilities in the nonlinear regime coexist and affect each other [15] and therefore it is important to investigate and understand the interplay among various instabilities. In light of considerable current interest in self-focusing and SRS, lot of work has already been done in the past [16–26]. In most of the above mentioned works, investigations have been carried out in the paraxial approximation [27,28] due to small divergence angles of the laser beams involved. Also, if the beam width of laser beam used is comparable to the wavelength of the laser beam, paraxial approximation is not valid. In some experiments, where solid state lasers are used, wide angle beams are generated for which the paraxial approximation is not applicable and therefore one cannot accurately compare the theoretical results with that of experimental results. Moreover, in paraxial theory non-linear part of the dielectric constant is taylor expanded up to second order term and higher order terms are neglected. However, moment theory [29,30] is based on the calculation of moments and does not suffer from this defect. In moment theory approach, non-linear part of the dielectric constant is taken as a whole in calculations [31–37]. Moment theory is difficult to apply, wherever the propagation of more than one wave is involved and therefore one always prefer to apply paraxial theory, in which the mathematical calculations become simpler as compared to moment theory approach. To the best of our knowledge, so far no one has

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used moment theory approach to study the SRS. So, the novelty of the present work is that instead of paraxial part,the full non-linear part of the dielectric constant has been taken in to account in the present investigation and therefore the approach is more realistic.

In the present paper, SRS of a Gaussian laser beam from a collisionless plasma has been investigated. The pump wave (ω_0, k_0) interacts with pre-excited electron plasma wave (ω, k) to generate a scattered wave ($\omega_0-\omega$, k_0-k). As a specific case, back scattering for which $k\simeq 2k_0$ has been discussed. The pump beam exerts a ponderomotive force on the electrons, leading to redistribution of carriers and consequently, the pump beam becomes self-focused. The dispersion relation for electron plasma wave is also significantly modified. The phase velocity of the electron plasma wave becomes minimum on the axis and increases away from it. Therefore, if appropriate conditions are satisfied, the electron plasma wave may also get focused. Since the scattered intensity is proportional to the intensities of the pump and electron plasma wave, it is therefore expected that the self-focusing should lead to enhanced back-scattering.

The paper is organized as follows: Section 2 is devoted to solution of wave equation for the pump beam and derivation of beam width parameter of pump beam with the help of moment theory approach. Section 3 is devoted to solution of wave equation for electron plasma wave and derivation of beam width parameter of electron plasma wave. In Section 4, the wave equation for the back-scattered wave is solved by moment theory approach and differential equation for beam width parameter of back-scattered wave is derived. Expression for reflectivity 'R' of the back-scattered is also derived. Finally a detailed discussion of results is presented in Section 5.

2. Solution of wave equation for pump beam

Consider the propagation of Gaussian laser beam of frequency ω_0 and wave vector k_0 in hot collisionless and homogeneous plasma along z-axis. When a laser beam propagates through plasma, the transverse intensity gradient generates a ponderomotive force, which modifies the plasma density profile in the transverse direction as [28]

$$N_{0e}=N_{00}\exp\left[-\frac{3}{4}\alpha\frac{m}{M}E\cdot E^*\right] \tag{1}$$

where $\alpha=e^2M/(6k_BT_0\gamma m^2\omega_0^2)$. e and m are the electronic charge and mass, M and T_0 are respectively the mass of ion and equilibrium temperature of plasma. N_{0e} is electron concentration in the presence of laser beam, N_{00} is the electron concentration in the absence of laser beam, k_B is Boltzman's constant and γ is ratio of two specific heats.

The initial intensity distribution of beam along the wavefront at $z=0$ is given by

$$E_0\cdot E_0^*|_{z=0}=E_{00}^2\exp\left[\frac{-r^2}{r_0^2}\right] \tag{2}$$

where $r^2=x^2+y^2$ and r_0 is the initial width of the pump beam and r is radial co-ordinate of the cylindrical co-ordinate system. Slowly varying electric field E_0 of the pump beam satisfies the following wave equation.

$$\nabla^2E_0-\nabla(\nabla\cdot E_0)+\frac{\omega_0^2}{c^2}\epsilon E_0=0 \tag{3}$$

In the Wentzal–Kramers–Brillouin (WKB) approximation, the second term $\nabla(\nabla\cdot E_0)$ of Eq. (3) can be neglected, which is justified, when $(c^2/\omega_0^2)|(1/\epsilon)\nabla^2\ln\epsilon|\ll 1$,

$$\nabla^2E_0+\frac{\omega_0^2}{c^2}\epsilon E_0=0 \tag{4}$$

where $\epsilon=\epsilon_0+\Phi(AA^*)$, ϵ_0 and $\Phi(AA^*)$ are linear and non-linear parts of the dielectric constant respectively, where $\epsilon_0=1-(\omega_p^2/\omega_0^2)$ and $\Phi(AA^*)=(\omega_p^2/\omega_0^2)[1-(N_{0e}/N_0)]$ and $\omega_p=\sqrt{4\pi N_0e^2/m}$ is the electron plasma frequency. Further taking E_0 in Eq. (4) as

$$E_0=A(r,z)\exp[\iota\{\omega_0t-k_0z\}] \tag{5}$$

where $A(r,z)$ is a complex function of its argument. The behavior of the complex amplitude $A(r,z)$ is governed by the parabolic equation obtained from the wave Eq. (4) in the WKB approximation by assuming variations in the z direction being slower than those in the radial direction,

$$\iota\frac{dA}{dz}=\frac{1}{2k_0}\nabla_\perp^2A+\chi(AA^*)A \tag{6}$$

where $\chi(AA^*)=(k_0/2\epsilon_0)(\epsilon-\epsilon_0)$ and $\epsilon=\epsilon_0+\Phi(|AA^*|)$.

Now from the definition of the second order moment, the mean square radius of the beam is given by

$$<a^2>=\frac{\int\int(x^2+y^2)AA^*\,dx\,dy}{I_0}. \tag{7}$$

From here one can obtain the following equation.

$$\frac{d^2<a^2>}{dz^2}=\frac{4I_2}{I_0}-\frac{4}{I_0}\int\int Q(|A|^2)\,dx\,dy \tag{8}$$

where I_0 and I_2 are the invariants of Eq. (6) [29]

$$I_0=\int\int|A|^2\,dx\,dy \tag{9}$$

$$I_2=\int\int\frac{1}{2k_0^2}(|\nabla_\perp|A|^2-F)\,dx\,dy \tag{10}$$

with [30]

$$F(|A|^2)=\frac{1}{k_0}\int\chi(|A|^2)d(|A|^2) \tag{11}$$

and

$$Q(|A|^2)=\left[\frac{|A|^2\chi(|A|^2)}{k_0}-2F(|A|^2)\right]. \tag{12}$$

For $z>0$, we assume an energy conserving Gaussian ansatz for the laser intensity [27,28]

$$AA^*=\frac{E_{00}^2}{f_0^2}\exp\left\{-\frac{r^2}{r_0^2f_0^2}\right\}. \tag{13}$$

From Eqs. (7), (9) and (13), it can be shown that

$$I_0=\pi r_0^2E_{00}^2, \tag{14}$$

$$<a^2>=r_0^2f_0^2 \tag{15}$$

where f_0 is the dimensionless beam width parameter and r_0 is the beam width at $z=0$. Now, from Eqs. (8) to (15), we get

$$\frac{d^2f_0}{d\xi^2}+\frac{1}{f_0}\left(\frac{df_0}{d\xi}\right)^2=\frac{2k_0^2}{\pi E_{00}^2f_0}\left[I_2-\int\int Q(|E_0|^2)\,dx\,dy\right] \tag{16}$$