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# Another way to shape the comprehensive analytical approach describing electromagnetic energy distribution through four-slab-layer structures

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## Abstract

Starting from the generalised four-slab layer in electromagnetic theory and photonics, this paper introduces a convenient method and a new proper change of variables in order to obtain the global analytical expressions of the power flows in such multilayer structures for the $TE_m$ and $TM_m$ optical modes. These proper changes of variables and relevant definitions of apt new parameters ($\Theta$, $W$, $Y$ and $\xi$) allow us to derive and shape new general analytical formulations and normalizations in terms of power flows. According to such specific parameters, it can be noted that such a comprehensive result brings in an effective criteria form of the classical results ascribed to three-slab problems. Moreover, we have verified with specific cases regarding three-slab problems the validity of our new global frame for analysing power flows. It clearly appears that classical three-slab-waveguide expressions directly stem from our formulation. Naturally, this global four-slab-waveguides approach can be used directly to the analytical calculus of corresponding ratios of power between the different layers, such as the core compared with buffer layers as upper and lower claddings.


***Keywords:*** Electromagnetic theory; Multilayer structures; Power flow

## 1. Introduction

Theoretical studies of the field theory of guided waves [1] and integrated optics [2] play a key role in optical telecommunication [3,4] and micro-sensor systems [5,6]. An increasing number of such optical devices is based on multilayer waveguide structures [7] for chemical or biochemical applications. Considering multilayer-slab structures, it is necessary to optimize the opto-geometrical parameters in order to control the localisation of the evanescent wave, to increase the length of such evanescent probes to improve the sensor sensitivity, that is to act on the whole distribution of the energy of the appropriate optical modes regarding a given application.

Intended to unify, this paper introduces first the adequate variables, allowing us to obtain the prevalent analytical expressions of the power flows through any four-slab-multilayer structure for both $TE_m$ and $TM_m$ optical modes. In this formulation, as a main point the definitions of adequate and specific parameters allow us

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to advance the versatile analytical power-flow expressions that meet the well-known three-slab-waveguide effective criteria form. Moreover, thanks to such a choice of variables, the classical three-slab-waveguide expressions naturally appear to stem from our formulation. Then, such expressions are most suitable to calculate the corresponding ratios of power flows through the core and the upper and lower claddings in any four-slab waveguide.

## 2. Theoretical approach and analytical expressions of the power flow

### 2.1. Problem definition: global asymmetric four-slab-waveguide structures and theory

In integrated optics, the resolution and calculation of guided modes in most multilayer slab waveguides directly stem from Maxwell's equations [1,2]. Indeed, the classical propagation wave equation in each dielectric layer of such opto-geometrical structures can be solved by way of the proper and continuous boundary conditions of the electric and magnetic fields at the interfaces: it is actually possible to determine, respectively, the modal or effective propagation constants from the eigenvalue equation for guided modes and the spatial field distributions related to the eigenvectors of such physical systems [8].

Consider the generic four-slab-waveguide structure as shown in Fig. 1; the opto-geometric parameters of such a physical system are the dielectric distribution related to the index profile ($n_i$, $i = 1$–$4$), the free propagation constant $k_0$ (or the wavelength $\lambda_0$), and the thicknesses of the core waveguide and the first upper-cladding layer, $2h$ and $2d$. The normalized quantities $q$, $t$, $p$ and $r$ hinge on the above parameters together with the modal or effective propagation constant $\beta \equiv \beta_{\mathrm{eff}} = k_0 n_{\mathrm{eff}}$. The guided modes in such an optical multilayer (called $\mathrm{TE}_m$ and $\mathrm{TM}_m$ polarisations, with the $m$ integer standing for the quantification due to the $x$-direction's confinement) are assumed to propagate in the $z$-direction with a phase term given as $\exp[\mathrm{j}(\omega t - \beta z)]$. The design illustrated in Fig. 1 refers to a waveguide structure with $n_3 \leqslant n_4 \leqslant n_2 \leqslant n_1$. Such an adequate approach allows us to include any other distribution regarding the $n_i$ values indices. As shown in Fig. 1, two different families of solutions for guided modes can be defined relative to their $m$ quantification into both light cones: (i) $k_0 n_2 \leqslant \beta \leqslant k_0 n_1$ and (ii) $k_0 n_3 \leqslant \beta \leqslant k_0 n_2$. First inequality (i) deals with the optical family modes involving the upper value of effective propagation constants $\beta$ (or effective indices $n_{\mathrm{eff}}$) depicting a strong confinement of their spatial field distributions. In the rest of the paper, in order to handle both families of guided modes into a single expression, we will resort to a double notation regarding all following equations and expressions; the upper notations $|^{\bullet}$ account for the first family modes or (i) conditions, whereas the lower notations $|_{\bullet}$ submit to the second family or (ii) prior conditions. Considering such global four-slab waveguides (Fig. 1) the $x$-spatial distribution of the optical modes ($E_y$ for TE polarisation, and $H_y$ for TM polarisation, Fig. 1) may be expressed as

$$\begin{cases} \dfrac{B \left| \begin{matrix}\cosh\\ \cos\end{matrix} \right. (2dt+\chi)}{\left| \begin{matrix}\cosh\\ \cos\end{matrix} \right. (\chi)} \exp[r(2d-x)], & x \geqslant 2d \\ \dfrac{B \left| \begin{matrix}\cosh\\ \cos\end{matrix} \right. (tx+\chi)}{\left| \begin{matrix}\cosh\\ \cos\end{matrix} \right. (\chi)}, & 0 \leqslant x \leqslant 2d \\ A \sin(qx) + B\cos(qx), & -2h \leqslant x \leqslant 0 \\ [-A \sin(2hq) + B\cos(2hq)]\exp[p(x+2h)], & x \leqslant -2h \end{cases}, \tag{1}$$

where the quantities $r = (\beta^2 - k_0^2 n_3^2)^{1/2}$, $t = [|\pm(\beta^2 - k_0^2 n_2^2)]^{1/2}$, $q = (k_0^2 n_1^2 - \beta^2)^{1/2}$, $p = (\beta^2 - k_0^2 n_4^2)^{1/2}$ are positive (Fig. 1), and $A$, $B$, $\chi$ integration constants with respect to the classical resolution of the differential wave equation observed in each layer.

It is clear that such modal expressions $E_y$ or $H_y$ include the continuous boundary conditions between adjacent layers at $x = 2d$, 0 and $-2h$. The usual boundary-value technique can be applied considering the two continuous boundary conditions regarding $x = 0$, $x = 2d$, related to the other components of the fields inferred from the Maxwell's equations $H_z = (\mathrm{j}/\omega\mu_0)\,(\partial E_y/\partial x)$ for TE modes ($E_y$) and $E_z = (\mathrm{j}/\omega\varepsilon_0 n_i^2)\,(\partial H_y/\partial x)$ for TM modes ($H_y$); hence, it is possible to define two sets of relations matching both integration constants $A$ and $B$:

$$\frac{A}{B} = |\pm\eta_{1\to2}\left(\frac{t}{q}\right)\left| \begin{matrix} th(\chi) \\ tg(\chi) \end{matrix} \right. \quad \text{or} \quad \chi = |\pm \left| \begin{matrix} th^{-1} \\ tg^{-1} \end{matrix} \right. \left[\frac{A}{B}\left(\frac{q}{t}\right)\eta_{2\to1}\right], \tag{2}$$

and

$$\left| \begin{matrix} th \\ tg \end{matrix} \right. (2\,\mathrm{d}t+\chi) = |\mp\eta_{2\to3}\left(\frac{r}{t}\right) \quad \text{or} \quad \chi = \left| \begin{matrix} -th^{-1} \\ tg^{-1} \end{matrix} \right. \left(\eta_{2\to3}\frac{r}{t}\right) - 2\,\mathrm{d}t. \tag{3}$$

As a result

$$\frac{A}{B} = |\pm\eta_{1\to2}\left(\frac{t}{q}\right)\left| \begin{matrix} th\left[-th^{-1}\left(\eta_{2\to3}\left(\frac{r}{t}\right)\right) - 2\,\mathrm{d}t\right] \\ tg\left[tg^{-1}\left(\eta_{2\to3}\left(\frac{r}{t}\right)\right) - 2\,\mathrm{d}t\right] \end{matrix} \right.$$

according to the said notation, $\eta_{i\to j} = 1$ for the TE modes, and $(n_i/n_j)^2$ for the TM modes ($i, j = 1$–$4$).