Optik 124 (2013) 5289–5292

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Error compensation for mirror symmetry absolute test

Yuhang He[a,*], Liqun Cai[a], Qiang Li[a], Bo Gao[a], Yiping Cao[b]

[a] Chengdu Fine Optical Engineering Research Center, Chengdu, Sichuan 610091, China
[b] Sichuan University, Chengdu, Sichuan 610064, China

ARTICLE INFO

Article history:
Received 10 September 2012
Accepted 24 January 2013

Keywords:
Absolute test
Measurement accuracy
Error compensation
Mirror symmetry
Zernike polynomials

ABSTRACT

A method is proposed to compensate intrinsic error in mirror symmetry absolute test. Because of the limitation of rotation times of flat A, intrinsic error of $CN\theta$ terms occurs in reconstructed wavefronts of flats A, B and C. If flat A is rotated to a new azimuthal position, and the wavefront difference between two measurements before and after rotation is calculated, the Zernike coefficients of $CN\theta$ terms can be obtained by solving coefficient equations due to rotation invariability of the form of Zernike polynomials in polar coordinates. Therefore, the intrinsic error of $CN\theta$ terms may be compensated. Because the amount of $CN\theta$ terms is infinite, the compensated terms are decided in terms of the balance between intrinsic error reduction and computational effort. Computer simulation proves the validity of the proposed method.

1. Introduction

In interferometric optical testing, an optical surface with higher precision is needed as fiducial reference if the tested surface is of great precision. Testing accuracy of interferometric metrology is generally conditioned by the precision of the reference surface. In order to solve the problem of fiducial reference, an early idea is to use a liquid-surface as the reference surface because of its curvature radius equal to the earth surface [1]. However, the disadvantages include it is easy to be disturbed by dust, mechanical vibration, capillary action, temperature grads, magnetic force effect from outside and so on. In addition, the tested optic must be mounted in horizontal posture, which leads to a sag due to gravity effect, especially for those with great size. In fact, the method is difficult to be applied in testing practice.

A different idea is to use so-called absolute tests, which separate error of interferometer reference wavefront from that of tested surface. The best known of the absolute tests is the three-flat test proposed by Schulz and Schwider [2,3]. The classical three-flat test could only determine one linear profile from three setups or, with a 180° rotation of one of the flats, two linear profiles from four setups. There have been many efforts to extend the classical three-flat test for profiles to full-aperture topography [4–16]. The typical approaches include Zernike polynomial fitting method by Fritz [6], even and odd function method by Ai [8], rotation symmetry method by Evans [10], mirror symmetry method by Griesmann [14] and so on. Of them, mirror symmetry method is believed as one of the simplest and most efficient methods because of simple measure-data analysis, minimal computational effort, and high accuracy.

In mirror symmetry absolute testing, one of the three flats should be rotated theoretically infinite times by equal azimuthal interval in one period of 360°, which cannot be implemented in practice. If the mount of rotation times is N, there are intrinsic errors of Zernike terms of angular order $CN\theta$ for each flat wavefront. Therefore if the $CN\theta$ terms can be obtained, the wavefronts of three flats will be compensated.

2. Principle of mirror symmetry method

In classical three-flat test, three flats A, B, C can be compared in pairs using the measurement sequence (BA, CA, CB) with a Fizeau interferometer as shown in setups 1, 3, and 4 of Fig. 1. Because flat B are needed as the reference plane in setup 1 and the tested plane in setup 4, only one linear profile at $x=0$ is determined from three setups, which is called the well-known three-flat problem. It was proved many years ago that the three-flat problem cannot be solved by comparing more than three flats in the test. In mirror symmetry method, a group of measurements of reference flat B against test flat A are added as shown in setup 2 of Fig. 1, in which A is rotated $N-1$ times by the angular interval $\Delta\phi=2\pi/N$. The wavefront $W_2(x, y)$ can be realized by averaging N measurement results, thus removing the rotationally variant component of flat A, which is expressed as following [14]:

$$W_2(x,y) = \frac{1}{N}\sum_{k=0}^{N-1}\{W_B(-x,y) + [W_A(x,y)]^{k\Delta\phi}\} \approx W_B(-x,y) + W_A^R(x,y) \quad (1)$$

* Corresponding author.
E-mail address: hang_yu_he@163.com (Y. He).

0030-4026/$ – see front matter © 2013 Elsevier GmbH. All rights reserved.
http://dx.doi.org/10.1016/j.ijleo.2013.01.121

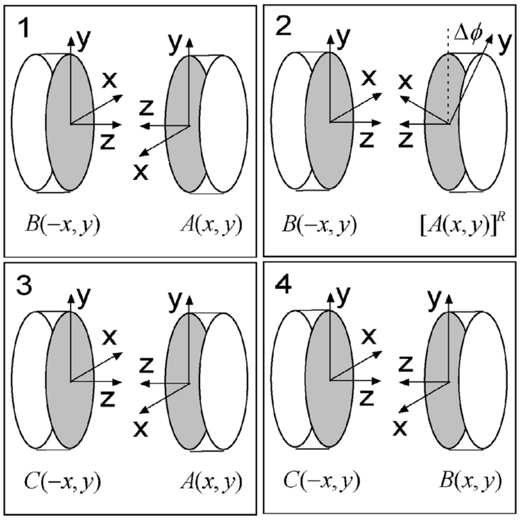

Fig. 1. Measurement sequence of mirror symmetry.

The operator $[\bullet]^{k\phi}$ is used to indicate a rotation by an angle $k\phi$. W_A^R is the rotation invariant part of W_A. The measurement sequence shown in Fig. 1 corresponds to the flat test equation:

$$\begin{bmatrix} W_1(x,y) \\ W_2(x,y) \\ W_3(x,y) \\ W_4(x,y) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} W_A(x,y) \\ W_B(x,y) \\ W_B(-x,y) \\ W_C(-x,y) \\ [W_A(x,y)]^R \end{bmatrix} \tag{2}$$

The vectors in this equation can be split into even and odd components. The even and odd components of wavefronts A, B and C can be obtained by separately solving even and odd component equations, subsequently which can be added to acquire the whole wavefronts of three flats. The wavefront results can be expressed as the following equation [14]:

$$\begin{bmatrix} W_A \\ W_B \\ W_C \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 & -1 & 2 & -2 & 0 & 0 \\ 1 & -1 & 1 & 2 & -2 & -2 & 2 \\ -1 & 1 & 1 & 2 & -2 & -2 & 0 \end{bmatrix} \begin{bmatrix} W_1^e \\ W_3^e \\ W_4^e \\ W_1^o \\ W_2^o \\ W_3^o \\ W_4^o \end{bmatrix} \tag{3}$$

Operator $[\bullet]^e$ indicates even component, and $[\bullet]^o$ odd component. For wavefront $W(x, y)$, even and odd components are expressed separately as

$$W^e(x,y) = \tfrac{1}{2}[W(x,y) + W(-x,y)] \tag{4}$$

$$W^o(x,y) = \tfrac{1}{2}[W(x,y) - W(-x,y)] \tag{5}$$

Mirror symmetry method provides a pixel-by-pixel solution to three-flat problem, which only needs simple mirror operation at y axis on the measurement wavefronts, thus requiring minimal computational effort.

3. Error compensation method

3.1. Intrinsic error analysis

For $W_A^R(x,y)$ in Eq. (1), it should be written as

$$W_A^R(x,y) = \lim_{N\to\infty} \left\{ \frac{1}{N} \sum_{k=0}^{N-1} [W_A(x,y)]^{k\Delta\phi} \right\} \tag{6}$$

That is to say that only if the measurement amount of N is infinite, it occurs that

$$W_2(x,y) = W_B(-x,y) + W_A^R(x,y) \tag{7}$$

If N is limited and $W_2(x,y)$ in Eq. (1) is substituted into Eq. (3), error will be generated. In fact, if N is a limited amount, it occurs that [10,14]

$$W_2(x,y) = W_B(-x,y) + W_A^R(x,y) + \Omega_A^{cN\theta}(x,y) \tag{8}$$

$\Omega_A^{cN\theta}$ is a part of rotation variance of wavefront A, and c represents positive integers. If a wavefront function is fitting to Zernike polynomials, $\Omega_A^{cN\theta}$ corresponds to the terms of angular order $CN\theta$. Therefore if only $\Omega_A^{cN\theta}$ is found, and W_2 in Eq. (3) is replaced by $W_2 - \Omega_A^{cN\theta}$, the intrinsic error due to the limited rotation times can be eliminated.

3.2. Error compensation

A wavefront function W defined in unit circle domain can be expressed in the form of Zernike polynomials as [17]

$$W(r,\theta) = \sum_{n,l} R_n^l [z_n^l \cos(l\theta) + z_n^{-l} \sin(l\theta)] \tag{9}$$

z_n^l and z_n^{-l} are the coefficients of Zernike polynomials. If W is rotated the angle of φ clockwise, the result is expressed as

$$[W(r,\theta)]^{\varphi} = W(r,\theta-\varphi) = \sum_{n,l} R_n^l [z_n^{\varphi,l} \cos(l\theta) + z_n^{\varphi,-l} \sin(l\theta)] \tag{10}$$

where $z_n^{\varphi,l}$ and $z_n^{\varphi,-l}$ are

$$z_n^{\varphi,l} = z_n^l \cos(l\varphi) - z_n^{-l} \sin(l\varphi) \tag{11}$$

$$z_n^{\varphi,-l} = z_n^{-l} \cos(l\varphi) + z_n^l \sin(l\varphi) \tag{12}$$

The difference E between W and W^{φ} can be expressed as

$$E(r,\theta) = W(r,\theta) - [W(r,\theta)]^{\varphi} = \sum_{n,l} R_n^l [e_n^l \cos(l\theta) + e_n^{-l} \sin(l\theta)] \tag{13}$$

Solving Eq. (9) to Eq. (13), it can be obtained that

$$z_n^l = \frac{1}{2}\left[e_n^l - \frac{e_n^{-l}\sin(l\varphi)}{1-\cos(l\varphi)} \right] \tag{14}$$

$$z_n^{-l} = \frac{1}{2}\left[e_n^{-l} + \frac{e_n^{l}\sin(l\varphi)}{1-\cos(l\varphi)} \right] \tag{15}$$

Therefore, the Zernike polynomial coefficients of $\Omega_A^{cN\theta}$ in Eq. (8) can be acquired if flat A is rotated a different angle in addition to the routine $N-1$ rotation. However φ is not equal to $2\pi/cN$, or the denominator of the right of Eqs. (14) and (15) would be zero. Because of the orthogonality characteristic of Zernike polynomials, the polynomial coefficients of $\Omega_A^{cN\theta}$ can be calculated by fitting only the terms of angular order $CN\theta$, not using the whole terms, which reduce the computational effort.