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A new three-dimensional chaotic system with wide range of parameters

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ABSTRACT

A new three-dimensional chaotic system with large scope of parameters is proposed. Its properties such as equilibrium points, Poincaré map, the largest Lyapunov exponent spectra and bifurcation diagrams are analyzed theoretically and numerically. Theoretical analyses and simulation tests indicate that the proposed chaotic system can keep chaotic in a wide range of parameters. The system is recommendable for many engineering applications such as secure communications, cryptology, information processing, etc.

1. Introduction

Chaos, known by its deterministic, unpredictability and extremely sensitive dependence on initial conditions, stems from nonlinear systems [1]. It can be applied in the field of data communications, signal detection, cryptology and so on. Since the Lorenz chaotic system [2] was proposed in 1963, many chaotic systems closely related to the Lorenz system had been reported, such as Rössler system [3], Chen system [4], Lü system [5], Liu system [6], etc. Lü et al. proposed a real four-scroll chaotic system in [7]. Since then, many chaotic systems generating two-scroll or multi-scroll attractors were subsequently proposed [8–17]. Table 1 shows the equations of the systems.

In engineering fields, the parameter range of chaotic systems is very important for applications. Usually, wider range of chaotic systems is preferred. In this paper, we propose a two-scroll chaotic system which can exhibit chaotic behaviors in a larger scope of parameters than all of the systems shown in Table 1. The proposed system with wide parameter range is desirable for such applications as secure communications, cryptography, information processing, etc.

This paper is organized as follows. Mathematic model, dynamical features of the new three-dimensional chaotic system are briefly introduced in Section 2. Dynamical behaviors such as equilibrium points, Poincaré map, the largest Lyapunov exponent spectra and bifurcation diagrams of the system are discussed in Section 3. Section 4 is the conclusions.

2. A new three-dimensional chaotic system

A new three-dimensional chaotic system is established by the following equations.

$$\begin{cases} \dot{x} = ax + dyz + gy^2 \\ \dot{y} = by + exz + hz \\ \dot{z} = cz + fxy \end{cases} \tag{1}$$

where the a, b, c, d, e, f, g, h are non-zero parameters of the system. Fig. 1 shows phase diagrams of the system (1) with different parameters. Fig. 2 shows the Poincaré map projected in the $\boldsymbol{y}$–z plane.

For system (1),

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = a + b + c$$

In order to ensure the dissipation of the system, it is required that $(a+b+c)<0$. Then the system can contract at an exponential rate $e^{(a+b+c)t}$. When $a=-3$, $b=5$, $c=-10$, the system converges at a rate of e^{-8t}. Therefore, each volume cell containing the trajectories of the system ultimately shrinks into zero as $t\to\infty$. In order to make the motion of the system (1) settle onto an attractor, $(a+b+c)<0$ should be satisfied.

3. Equilibrium points and stability

For calculating equilibria, let

$$\begin{cases} ax + dyz + gy^2 = 0 \\ by + exz + hz = 0 \\ cz + fxy = 0 \end{cases} \tag{2}$$

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Table 1
Equations of some chaotic systems with two-scroll or multi-scroll attractors.

Source	Equations	Attractors
Lü et al. [7]	$\begin{cases}\dot{x}=-\frac{ab}{a+b}x-yz\\ \dot{y}=ay+xz\\ \dot{z}=bz+xy\end{cases}$	Four-scroll
Zhou and Chen [8]	$\begin{cases}\dot{x}_1=a_1x_1+a_2x_2+x_2x_3\\ \dot{x}_2=b_2x_2-x_1x_3+b_{23}x_2x_3\\ \dot{x}_3=-x_1x_2+c_3x_3\end{cases}$	Four-scroll
Chen et al. [9]	$\begin{cases}\dot{x}=ax+ky-yz\\ \dot{y}=-by-z+xz\\ \dot{z}=-x-cz+xy\end{cases}$	Three-scroll and four-scroll
Li [10]	$\begin{cases}\dot{x}=Ax+By+Cxz\\ \dot{y}=Dx+Ey+Fxz\\ \dot{z}=Gx^2+Hxy+Iz\end{cases}$	Three-scroll
Wang [11]	$\begin{cases}\dot{x}=a(x-y)-yz\\ \dot{y}=-by+xz\\ \dot{z}=-cz+dx+xy\end{cases}$	Three-scroll and four-scroll
Wang et al. [12]	$\begin{cases}\dot{x}=ax+cyz\\ \dot{y}=bx+dy-xz\\ \dot{z}=ez+fxy\end{cases}$	Four-scroll
Buncha and Banlue [13]	$\begin{cases}\dot{x}=a(y-x)\\ \dot{y}=-xz\\ \dot{z}=-b+xy\end{cases}$	Two-scroll
Liu et al. [14]	$\begin{cases}\dot{x}=-ax-ey^2\\ \dot{y}=by-kxz\\ \dot{z}=-cz+mxy\end{cases}$	Two-scroll
Wang [15]	$\begin{cases}\dot{x}=-\frac{ab}{a+b}x-yz\\ \dot{y}=ay+xz\\ \dot{z}=bz+cx+xy\end{cases}$	Three-scroll and four-scroll
Li et al. [16]	$\begin{cases}\dot{x}_1=-ax_1+fx_2x_3\\ \dot{x}_2=cx_2-dx_1x_3\\ \dot{x}_3=-bx_3+ex_2^2\end{cases}$	Two-scroll
Dadras and Momeni [17]	$\begin{cases}\dot{x}=y-ax+byz\\ \dot{y}=cy-xz+z\\ \dot{z}=dxy-hz\end{cases}$	Two-scroll, three-scroll and four-scroll

$E_1(0, 0, 0)$ is one of the equilibria. For non-zero equilibria (x, y, z), we get

$$z=-\frac{f}{c}xy \tag{3}$$

$$efx^2+hfx-bc=0 \tag{4}$$

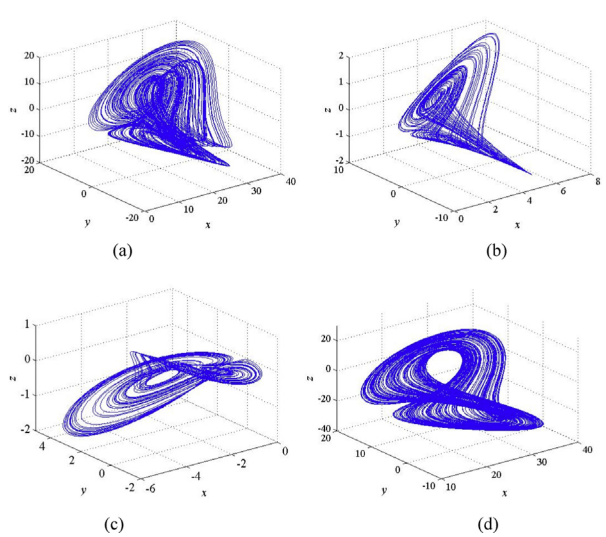

Fig. 1. Phase diagrams ($b=5$, $c=-10$, $d=f=1$, $e=-1$): (a) $a=-3$, $g=1$, $h=16$; (b) $a=-3$, $g=1$, $h=-16$; (c) $a=-3$, $g=-2$, $h=16$; and (d) $a=-5$, $g=1$, $h=16$.

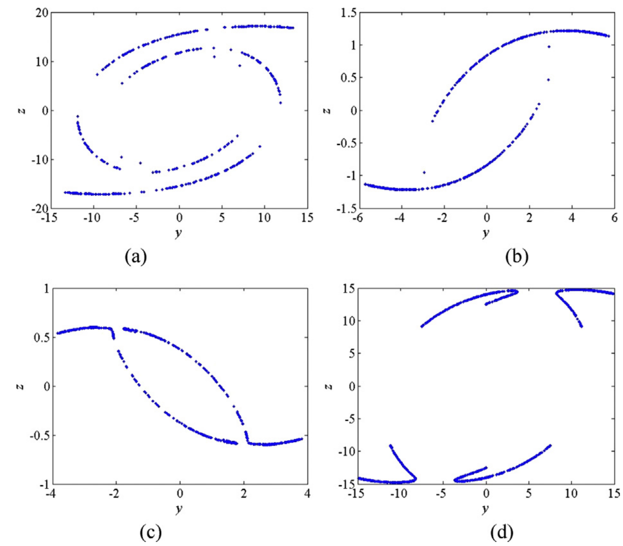

Fig. 2. Poincaré map projected in the y–z plane ($b=5$, $c=-10$, $d=f=1$, $e=-1$): (a) $a=-3$, $g=1$, $h=16$, $x=30$; (b) $a=-3$, $g=1$, $h=-16$, $x=4$; (c) $a=-3$, $g=-2$, $h=16$, $x=-3$; and (d) $a=-5$, $g=1$, $h=16$, $x=25$.

Then,

$$x_{1,2}=\frac{1}{2ef}(-hf\pm\sqrt{\Delta}) \tag{5}$$

where $\Delta=h^2f^2+4bcef$.

According to Eqs. (2) and (3), if $x\neq cg/df$, we have

$$y^2=\frac{acx}{dfx-cg} \tag{6}$$

1) If $\Delta<0$, the system has only one equilibrium points $E_1(0, 0, 0)$. If $bcef>0$, this case does not exist.
2) If $\Delta=0$ and $ach/(dfh+2ceg)>0$, the system has three equilibrium points $E_1(0, 0, 0)$, $E_2(-h/2e, \sqrt{ach/(dfh+2ceg)}, fh/2ce\sqrt{ach/(dfh+2ceg)})$, $E_3(-h/2e, -\sqrt{ach/(dfh+2ceg)}, -fh/2ce\sqrt{ach/(dfh+2ceg)})$. If $bcef>0$, this case does not exist either.
3) If $\Delta>0$, two different solutions x_1 and x_2 can be obtained according to Eq. (5). When $x_1\neq cg/df$, let $\xi=acx_1/(dfx_1-cg)$; When $x_2\neq cg/df$, let $\eta=acx_2/(dfx_2-cg)$.

If $\Delta>0$, the equilibrium points are shown in Table 2.

Table 2
Equilibrium points ($\Delta>0$).

Parameters	Equilibrium points
$\xi<0$, $\eta<0$	$E_1(0, 0, 0)$
$\xi>0$, $\eta<0$	$E_1(0, 0, 0)$, $E_2(x_1, \sqrt{\xi}, -fx_1\sqrt{\xi}/c)$, $E_3(x_1, -\sqrt{\xi}, fx_1\sqrt{\xi}/c)$
$\xi>0$, $x_2=cg/(df)$	$E_1(0, 0, 0)$, $E_2(x_1, \sqrt{\xi}, -fx_1\sqrt{\xi}/c)$, $E_3(x_1, -\sqrt{\xi}, fx_1\sqrt{\xi}/c)$
$\xi<0$, $\eta>0$	$E_1(0, 0, 0)$, $E_2(x_2, \sqrt{\eta}, -fx_2\sqrt{\eta}/c)$, $E_3(x_2, -\sqrt{\eta}, fx_2\sqrt{\eta}/c)$
$x_1=cg/(df)$, $\eta>0$	$E_1(0, 0, 0)$, $E_2(x_2, \sqrt{\eta}, -fx_2\sqrt{\eta}/c)$, $E_3(x_2, -\sqrt{\eta}, fx_2\sqrt{\eta}/c)$
$\xi>0$, $\eta>0$	$E_1(0, 0, 0)$, $E_2(x_1, \sqrt{\xi}, -fx_1\sqrt{\xi}/c)$, $E_3(x_1, -\sqrt{\xi}, fx_1\sqrt{\xi}/c)$, $E_4(x_2, \sqrt{\eta}, -fx_2\sqrt{\eta}/c)$, $E_5(x_2, -\sqrt{\eta}, fx_2\sqrt{\eta}/c)$