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Entanglement of a two two-level atom interacting with electromagnetic field in the presence of converter terms

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ABSTRACT

The exact solution of the problem of two two-level atoms interacting with a two-mode radiation field in the presence of a parametric converter term is presented. More precisely we have considered a Hamiltonian model that includes two types of interaction: one is the field–field (frequency converter type) and the other is the atom–field interaction. By introducing a canonical transformation an exact solution of the wave function in the Schrödinger picture is obtained. The result presented in this context is used to discuss the atomic inversion as well as the entropy squeezing and variance squeezing phenomena. We use the von Neuman entropy to measure the degree of entanglement between the atom and the field. These aspects are sensitive to changes in the converter parameter.

1. Introduction

The entanglement plays a central role in quantum information, quantum computation and communication, and quantum cryptography [1,2]. This is quite obvious from the efforts which have been devoted to characterize entanglement properties qualitatively and quantitatively and to apply them in quantum information. As one can see different schemes are proposed for many-particle entanglement generation. The simplest one of such schemes is to investigate the atom–field entanglement in the Jaynes–Cummings model (JCM) [3]. The model as well known describes an interaction of a two-level atom with a single mode quantized radiation field. In fact the model is of fundamental importance in the field of quantum optics [4,5] which is realizable to a good approximation in experiments with Rydberg atoms in high-*Q* superconducting cavities [6]. Also the model predicts a variety of interesting phenomena such as the quantum collapses and revivals [7] as well as atom–field entanglement [8], etc. The generalizations of the JCM for the two-atom case have also attracted considerable interest [9–11] where the entanglement between the atom and the field has been studied assuming that the cavity field is initially in the coherent state [8,12]. It would be interesting to point out that the two-photon interaction of a single two-level atom with the quantized electromagnetic field has also a considerable interest. [13]. This in fact is due to its importance in revealing the nonclassical properties of multiphoton transitions of atoms and the experimental development of a two-photon micromaser [14]. We can also see that the two-photon JCM is relevant to the study of the coupling between a single atom and a single-mode cavity field, where the two-photon transition is occurred. Here we may mention that, the cavity field spectra for a two photon JCM is also considered, however, in the presence of the Stark shift and the intensity-dependent coupling, for more details see Ref. [15]. There is no doubt the construction of the first single-mode two-photon laser by Gauthier et al. based on the dressed atom states which leads to the realization of the two-mode two-photon laser [16], opened the door for one to generalize the JCM in different directions. As a natural generalization of such a model is to increase the number of the cavity field modes or to increase the number of atoms. In fact when the number of atoms increases up to two, then the observation of radiation trapping is reported. However, the observation of the phenomenon of superradiance can be seen when the number of atoms increases more than two. This stimulated and encouraged many authors to study the problem of two two-level atoms and to take it into different directions. For instance the authors of Refs. [17–19] considered a two-atom Raman coupled model interacting with the two quantized cavity fields. They examined the atomic population dynamics

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as well as the photon statistics. The authors have been using the Hamiltonian model

$$\frac{\hat{H}}{\hbar} = \sum_{j=1}^{2}\left(\omega_j \hat{a}_j^{\dagger}\hat{a}_j + \frac{1}{2}\Omega_j \hat{S}_z^{(j)} + \lambda_j(\hat{S}_-^{(j)}\hat{a}_1\hat{a}_2^{\dagger} + \hat{S}_+^{(j)}\hat{a}_1^{\dagger}\hat{a}_2)\right), \tag{1}$$

where ω_j, j = 1, 2 are the fields frequency and λ_j is the interaction coupling parameters, while Ω_j, j = 1, 2 are the atomic frequency of the energy level difference. $\hat{a}_j^{\dagger}(\hat{a}_j), j = 1, 2$ are the creation and annihilation operators satisfy the commutation relation

$$[\hat{a}_i, \hat{a}_j^{\dagger}] = \delta_{ij}, \quad \delta_{ij} = \begin{vmatrix} 1, & i=j \\ 0, & i \neq j \end{vmatrix}, \tag{2}$$

while $\hat{S}_+^{(i)}(\hat{S}_-^{(i)})$ and $\hat{S}_z^{(j)}$ are the rising and lowering as well as the inversion operators. Furthermore, the problem of two two-level atoms interact with two nondegenerate parametric converter of a quantum electromagnetic field in an ideal cavity is also considered, see Refs. [20–22]. The models are used to describe such a quantum system is given by the Hamiltonians

$$\frac{\hat{H}}{\hbar} = \sum_{j=1}^{2}\left(\omega_j \hat{a}_j^{\dagger}\hat{a}_j + \frac{1}{2}\Omega_j \hat{S}_z^{(j)} + i\lambda_i(\hat{S}_-^{(j)}\hat{a}_1^{\dagger}\hat{a}_2 + \hat{S}_+^{(j)}\hat{a}_1\hat{a}_2^{\dagger})\right), \tag{3}$$

where the operators are involved in this equation have the same meaning as the operators in Eq. (1). Recently, much attention has been focused on the properties of the entanglement between field and atom, in particular, the entropy of the system. Knight and co-worker [23] have shown that entropy is a very useful operational measure of the purity of the quantum state, that automatically includes all moments of the density operator. The time evolution of the field (atomic) entropy reflects the time evolution of the degree of entanglement between the atom and the field. The higher entropy is the greater entanglement. The information concerning the field is usually inferred by a measurement of atomic properties. In an ideal cavity, the atom undergoes either one or a two-photon transition. The field entropy for the entangled state of a single two-level atom interacting with a single electromagnetic field has been studied [24]. However, the results have been obtained only for the case in which the effect of the Stark shift is ignored. In order to make the two-photon processes closer to the experimental realization, one should take into account the effect of the dynamic Stark shift in evaluating the field entropy [25]. In a previous communication we have introduced a Hamiltonian model consisting of two fields injected simultaneously within a perfect cavity to interact with a single atom. The interaction between the fields has been taken into account and considered to be in the parametric converter form [26]. We have ignored the effect of both Stark shift and Kerr-like medium and discussed some statistical properties of the system. We concentrated on the atomic inversion, the photon number distribution, the squeezing phenomenon, the correlation function and phase distribution. The Hamiltonian model used to describe such a system is given by

$$\frac{\hat{H}}{\hbar} = \sum_{j=1}^{2}(\omega_j \hat{a}_j^{\dagger}\hat{a}_j + i\lambda_j(\hat{S}_+ + \hat{S}_-)(\hat{a}_1^{\dagger}\hat{a}_2 - \hat{a}_1\hat{a}_2^{\dagger})) + \frac{1}{2}\Omega_0\hat{S}_z + \lambda_3(\hat{a}_1^{\dagger}\hat{a}_2 + \hat{a}_1\hat{a}_2^{\dagger}). \tag{4}$$

As one can see the model consists of a single two-level atom, however the main purpose of the present work is to consider the case in which the number of atoms is two. Therefore we have to modify the Hamiltonian to assume the form

$$\frac{\hat{H}}{\hbar} = \sum_{j=1}^{2}\left(\omega_j \hat{a}_j^{\dagger}\hat{a}_j + \frac{1}{2}\Omega_j \hat{S}_z^{(j)} + i\lambda_j(\hat{S}_+^{(j)} + \hat{S}_-^{(j)})(\hat{a}_1^{\dagger}\hat{a}_2 - \hat{a}_1\hat{a}_2^{\dagger})\right) + \lambda_3(\hat{a}_1^{\dagger}\hat{a}_2 + \hat{a}_1\hat{a}_2^{\dagger}). \tag{5}$$

By making a comparison between this model and the Hamiltonian (3) it is easy to realize that there is an extra term multiplied by the coupling parameter λ_3. This term is of the parametric converter form and represents the interaction between the fields themselves. This means that during the interaction operation a part of the field does not interact with the atoms which leads to an exchange of the fields energy with the atoms.

It should be noted that in the above Hamiltonian we have taken the couplings λ_1 and λ_2 to be constants. Thus the problem would be much easier and the chance, to find the dynamical operators or the wave function, becomes more tractable than for the non-conservative case. However, the existence of the nonlinearity in the interaction part would lead to some complications. Therefore, to avoid such complication we shall invoke a canonical transformation to remove the degenerate parametric term. This is done in the forthcoming section where the exact solution of the wave function in the Schrödinger picture is also obtained. In Section 3 we discuss the atomic inversion, while in Section 4 the mathematical form for the field entropy and we use numerical computations to examine the effect of difference of the photon numbers on the evolution of the field entropy and hence entanglement between the atom and the field. We devote Section 5 to discuss the entropy and the variance squeezing phenomena. Finally we give our conclusion in Section 6.

2. The time evolution operator

As we have mentioned above our aim of the present work is to examine some properties of the system described by the Hamiltonian model (1). Therefore we devote this section to find the tools to reach our goal. For this reason let us introduce the canonical transformation

$$\hat{a}_1 = \hat{b}_1 \cos\zeta + \hat{b}_2 \sin\zeta, \quad \hat{a}_2 = \hat{b}_2 \cos\zeta - \hat{b}_1 \sin\zeta \tag{6}$$

where the operators $\hat{b}_i(\hat{b}_i^{\dagger})$, $i = 1, 2$ have the same meaning of the operators $\hat{a}_i(\hat{a}_i^{\dagger})$, i = 1, 2 .while ζ is the rotation angle and will be determined later. It is easy to realize that the canonical transformation (3) always satisfy the conservation of total photon number law where

$$\hat{a}_1^{\dagger}\hat{a}_1 + \hat{a}_2^{\dagger}\hat{a}_2 = \hat{b}_1^{\dagger}\hat{b}_1 + \hat{b}_2^{\dagger}\hat{b}_2. \tag{7}$$

Which means that the sum of the photon number is invariant under this transformation. Now if we insert Eq. (6) into Eq. (1), then after some calculations we have

$$\frac{\hat{H}}{\hbar} = \sum_{i=1}^{2}\left(\overline{\omega}_j\hat{n}_j + \frac{\omega_0}{2}\hat{\sigma}_z^j + i\lambda_j(\hat{b}_1^{\dagger}\hat{b}_2\hat{\sigma}_+^j - \hat{b}_2^{\dagger}\hat{b}_1\hat{\sigma}_-^j)\right),$$
$$\hat{n}_j = \hat{b}_j^{\dagger}\hat{b}_j, \; j = 1, 2 \tag{8}$$

where the augmented frequencies are

$$\begin{aligned} \overline{\omega}_1 &= \omega_1\cos^2\zeta + \omega_2\sin^2\zeta - \lambda_2 \sin 2\zeta, \\ \overline{\omega}_2 &= \omega_2\cos^2\zeta + \omega_1\sin^2\zeta + \lambda_2 \sin 2\zeta, \\ \overline{\omega}_1 - \overline{\omega}_2 &= (\omega_1 - \omega_2)\cos 2\zeta - 2\lambda_2 \sin 2\zeta \end{aligned} \tag{9}$$