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Research on the ionospheric ion velocity distribution and the incoherent scattering spectra of the radar with the 16-moments approximation for the relaxation collision model

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ABSTRACT

The distribution function which given by Grad cannot be used to describe the ionospheric ion velocity distribution due to the larger anisotropic temperature appears in the high-latitude ionosphere. In this article, based on the Boltzmann equation, the relaxation collision model (RCM) was used to substitute the Boltzmann collision integration, and a non-Maxwell ion velocity distribution function with the 16-moments approximation for the bi-Maxwell distribution was given, and the ion transport equation with the 16-moments approximation was also derived and solved. Moreover, the ion velocity distribution, the solution of transport equation and the incoherent scattering spectra with the 13-moments and 16-moments approximation for the relaxation collision model were simulated, analyzed and compared. The research shows that compared with the 13-moments approximation, the 16-moments approximation with the bi-Maxwell distribution is more suitable to describe the characteristics of the anisotropic temperature ion distribution in the high-latitude ionosphere.

1. Introduction

The ionosphere is a weakly ionized plasma in the high-latitude, the collision frequency between the ions is less than collision frequency between the ion and neutral component, and it is also less than the ion gyromagnetic frequency, which means that the ion velocity distribution will significantly deviated from the Maxwell distribution form with the thermal equilibrium state. The ion velocity distribution function with the non-thermal equilibrium state in the auroral zone has been studied early by Cole [1], Schunk et al. [2–5], respectively, and they pointed out that the ion velocity distribution function can be expressed by an orthogonal polynomials and the core is to reasonably choose a weighting function (i.e., basis function). Moreover, the data from the satellite and incoherent scattering radar have also proved the existence of the non-Maxwell ion velocity distribution in the high-latitude ionosphere. Subsequently, a series of article by Hubert [6–8] showed that the convergence of the series will be accelerated if the ion velocity distribution function can be expanded with the bi-Maxwell distribution as the basis function and select a appropriate ion temperature under the medium electric field conditions, and this result is consistent with the experimental result of St. Maurice [3–5]. In addition, the transport equation of ion with the 16-moments approximation for the relaxation collisions model (RCM) and for the Maxwell molecular collision model (MMCM) was given by Barakat and Demars [9,10], respectively, and unfortunately they have not been able to solve it well. Moreover, the observation results which from the three stations EISCAT incoherent scattering radar by Perraut et al. [11] also shown that the ion velocity distribution function will have the bi-Maxwell distribution form when the ion temperature distribution shows a significant anisotropic characteristic in the high-latitude ionosphere, Meanwhile, the possible value of the ion anisotropic temperature was also given by Lovhaug and Fla [12], Moreover, the Monte Carlo method was also used to simulate the ion velocity distribution by Barghouthi [13]. In addition, Suvanto [14] calculated the incoherent scattering spectra with the non-thermal equilibrium plasma in the F layer, Hubert and Lathuillere [15] detailedly analyzed the effect of different ionospheric ion component on the incoherent scatter spectra. Moreover, Zheng and Wu et al. [16,17] try to modify the bi-Maxwell distribution function and given its analytical solution, but they cannot give a exact physical significance of it. Simultaneously, the ion incoherent scattering spectra was simulated with different moment approximation is given by Xu et al. [18–20], and Ma et al.

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[21] further studied the characteristics of the ion velocity distribution with 13-moments approximation in the high-latitude ionosphere. From the above analysis it can be seen that the effects of the stronger electric field, the collision frequency between the ion and neutral, as well as the anisotropic ion temperature distribution on the ion velocity distribution and its incoherent scattering spectra is still need to be further studied in the high-latitude ionosphere. In this article, the relaxation collision model was used to substitute the Boltzmann collision integration, and the non-Maxwell ion velocity distribution function was expressed by the 16-moments approximation for the bi-Maxwell distribution. Meanwhile, the ion velocity distribution, the solution of the ion transport equation and the ion incoherent scattering spectra for the relaxation collision model were simulated, analyzed and compared.

2. Polynomial solution of the ion velocity distribution function

The starting point for researching the ion velocity distribution is to solve the Boltzmann equation. Usually, for weakly magnetized plasma of the ionosphere, assuming that the particle number density (ions and electrons) is much smaller than the neutral components. Thus, for ion i, the form of the Boltzmann equation is presented as follows [22]

$$\frac{\partial f_i}{\partial t} + \boldsymbol{V}\cdot\nabla f_i + \left(\boldsymbol{G} + \frac{e_i}{m_i}\left(\boldsymbol{E} + \frac{1}{c}\boldsymbol{V}\times\boldsymbol{B}\right)\right)\cdot\nabla_v f_i = \frac{\delta f_i}{\delta t} \tag{1}$$

For convenience, we introduce a random speed c_i, that is

$$c_i = \boldsymbol{v} - \boldsymbol{u}_i \tag{2}$$

where $\boldsymbol{u}_i$ is the drift velocity of ion, thus, Eq. (1) can be expressed as

$$\frac{\partial f_i}{\partial t} + (\boldsymbol{c}_i + \boldsymbol{u}_i)\cdot\nabla f_i - \frac{D_i\boldsymbol{u}_i}{Dt}\cdot\nabla_{c_i} f_i - \boldsymbol{c}_i\cdot\nabla\boldsymbol{u}_i\cdot\nabla_{c_i} f_i + \left(\boldsymbol{G} + \frac{e_i}{m_i}\left(\boldsymbol{E} + \frac{1}{c}\boldsymbol{u}_i\times\boldsymbol{B}\right)\right)\cdot\nabla_{c_i} f_i + \frac{e_i}{m_i c}(\boldsymbol{c}_i\times\boldsymbol{B})\cdot\nabla_{c_i} f_i = \frac{\delta f_i}{\delta t} \tag{3}$$

where D_i/Dt is the convective derivative, it can be written as

$$\frac{D_i}{Dt} = \frac{\partial}{\partial t} + \boldsymbol{u}_i\cdot\nabla \tag{4}$$

In the Boltzmann equation, the term $\delta f_i/\delta t$ denotes the rate of change with time, and which caused by the collision in the given phase space, for the two-body elastic collision, the $\delta f_i/\delta t$ can be further expressed as

$$\frac{\delta f_i}{\delta t} = \int dc_t d\Omega g_{it}\sigma_{it}(g_{it},\theta)[f_i' f_t' - f_i f_t] \tag{5}$$

where $d\boldsymbol{c}_t$ is the integral region within the velocity space, $d\Omega$ is the solid angle, θ is the scattering angle, g_{it} is the relative velocity between ion i and t, and $\sigma_{it}(g_{it},\theta)$ is the differential scattering cross section.

2.1. Expanding by the Maxwell distribution

According to the Grad theory [23], the ionospheric ion velocity distribution function, f_i, can be approximately expanded into the following form by the zero-order distribution function $f_i^{(0)}$, that is

$$f_i = f_i^{(0)}\sum_r a_r(r)M_r(r, c_i) \tag{6}$$

where, a_r is the expansion coefficient, M_r denotes a series of complete orthogonal polynomials and $f_i^{(0)}$ denotes the zero-order ion velocity distribution function.

$$f_i^{(0)} = n_i\left(\frac{m_i}{2\pi k_B T_i}\right)^{3/2}\exp\left(-\frac{m_i c_i^2}{2k_B T_i}\right) \tag{7}$$

In Eq. (7), k_B is the Boltzmann constant, $\boldsymbol{c}_i$ expresses the random ion velocity and T_i denotes the ion temperature. Let the higher-order expansion coefficient is zero, then the coefficient of the reserved items can be expressed by the lower-order moment of the distribution function as

$$\boldsymbol{u}_i = \langle \boldsymbol{v}_i\rangle \quad \text{Ion drift velocity} \tag{8}$$

$$\boldsymbol{q}_i = \tfrac{1}{2}n_i m_i\langle c_i^2\boldsymbol{c}_i\rangle \quad \text{Heat flux vector} \tag{9}$$

$$\boldsymbol{p}_i = n_i m_i\langle \boldsymbol{c}_i\boldsymbol{c}_i\rangle \quad \text{Pressure tensor} \tag{10}$$

$$\boldsymbol{\tau}_i = \boldsymbol{P}_i - p_i\boldsymbol{I} \quad \text{Stress tensor} \tag{11}$$

$$\boldsymbol{\mu}_i = \tfrac{1}{2}n_i m_i\langle c_i^2\boldsymbol{c}_i\boldsymbol{c}_i\rangle \quad \text{High-order pressure tensor} \tag{12}$$

$$\boldsymbol{Q}_i = n_i m_i\langle \boldsymbol{c}_i\boldsymbol{c}_i\boldsymbol{c}_i\rangle \quad \text{Heat flux tensor} \tag{13}$$

$$T_{i\parallel} = m_i/k_B\langle c_{i\parallel}^2\rangle \quad \text{Parallel temperature} \tag{14}$$