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Discovering the generalized vectorial laws of multiple reflection and refraction

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ABSTRACT

This paper reports on the discovery of the generalized vectorial laws of multiple reflection and refraction as an extension of the earlier discovery of the generalized vectorial laws of reflection and refraction reported in 2005. The present discovery is novel and original and interesting from academic point of view. It will enrich the field of optics.

1. Introduction

This paper is concerned with the application of the generalized vectorial laws of reflection and refraction [1] discovered by the author in 2005. In Ref. [1], the traditional exposition of the reflection and refraction laws [2–13] has been reported to be ambiguous and to get rid of the ambiguity, the generalized vectorial laws of reflection and refraction have been developed [1]. The new vectorial portrayal of the reflection and refraction laws reported in Ref. [1] is much clearer leaving no room for confusion.

In Ref. [14], theoretical proof of the said discovered laws has been offered on the basis of the principle of conservation of momentum of photon. In Ref. [15], the said discovered laws have been employed to develop a lot of interesting physical insights to the mirror rotation problem as well as to the problem of rotation of refracting surface. The most remarkable fact that has been discovered in the said study is that the proposition "velocity of light is unbeatable" is not correct. Rather it is possible to have velocity exceeding the velocity of light – a result not in agreement with the Special theory of relativity. In Ref. [16], the generalized vectorial laws of reflection and refraction [1] have been employed to make an exhaustive study of reflection and refraction at spherical surfaces for various kinds of image formation.

As a further step forward, the generalized vectorial laws of reflection and refraction [1] have been applied in this paper to give birth to the generalized vectorial laws of multiple reflection and refraction. The running literature fall short of such laws and as a result, the generalized vectorial laws of multiple reflection and refraction offered in this paper must be claimed as novel and original, and they are sure to enrich the optical field.

2. Notations

i	Unit vector along the direction of incident ray
r	Unit vector along the direction of reflected ray
R	Unit vector along the direction of refracted ray
n	Unit vector along the conventional direction of the surface area of the reflecting or refracting surface at the point of incidence
I and J	Rectangular unit vectors

3. The discovered laws

In this section the laws reported in Ref. [1] are being presented.

3.1. The generalized vectorial law of reflection

If **i** and **r** represent unit vectors along the directions of the incident ray and reflected ray respectively and if **n** represents unit vector along the direction of the positive unit normal to the reflector at the point of incidence, then $\mathbf{n} \times \mathbf{i} = \mathbf{n} \times \mathbf{r}$.

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3.2. The generalized vectorial law of refraction

If **i** and **R** represent unit vectors along the directions of the incident ray and refracted ray of particular colour respectively and if **n** represents unit vector along the direction of the positive unit normal to the surface of separation at the point of incidence, then $\mathbf{n} \times \mathbf{i} = \mu(\mathbf{n} \times \mathbf{R})$, where μ = Refractive index of the second medium with respect to the first medium for the particular colour of light under consideration.

4. The generalized vectorial laws of multiple reflection and refraction

In this section, the aforesaid generalized vectorial laws of reflection and refraction will be employed to give birth to the generalized vectorial laws of multiple reflection and refraction.

4.1. The generalized vectorial law of multiple reflection

For 'p' number of successive reflection of a ray of light ($p > 1$), the generalized vectorial law of multiple reflection may be stated as

$$\mathbf{n_1} \times \mathbf{i_1} = \pm k(\mathbf{n_p} \times \mathbf{r_p})$$

where $\mathbf{i_1}$ = unit vector along the direction of the initial incident ray, $\mathbf{r_p}$ = unit vector along the direction of the reflected ray corresponding to the pth reflection, $\mathbf{n_1}$ = unit vector along the direction of the positive unit normal to the reflector corresponding to the first reflection, $\mathbf{n_p}$ = unit vector along the direction of the positive unit normal to the reflector corresponding to the pth reflection, 'k' is a positive constant depending exclusively on the angle of incidence at the first reflecting surface, being always equal to unity for successive reflection taking place at plane parallel mirrors and the +ve or −ve sign on the right hand side is to be taken depending on whether there exists even or odd number of rays, for each of which the two consecutive normal lie on opposite sides of such a ray.

The aforesaid generalized vectorial law of multiple reflection can be derived by considering the following two cases.

4.1.1. Case 1: When successive reflection take place in two plane parallel mirrors

Let us consider the successive reflections in two plane parallel mirrors as shown in Fig. 1. The generalized vectorial law of reflection for each of the five successive reflection shown in Fig. 1 may be written as follows.

$$\text{For first reflection}: \quad \mathbf{n_1} \times \mathbf{i_1} = \mathbf{n_1} \times \mathbf{r_1} \tag{1}$$

$$\text{For second reflection}: \mathbf{n_2} \times \mathbf{r_1} = \mathbf{n_2} \times \mathbf{r_2} \tag{2}$$

$$\text{For third reflection}: \mathbf{n_3} \times \mathbf{r_2} = \mathbf{n_3} \times \mathbf{r_3} \tag{3}$$

$$\text{For fourth reflection}: \mathbf{n_4} \times \mathbf{r_3} = \mathbf{n_4} \times \mathbf{r_4} \tag{4}$$

$$\text{For fifth reflection}: \mathbf{n_5} \times \mathbf{r_4} = \mathbf{n_5} \times \mathbf{r_5} \tag{5}$$

Now, since the two mirrors shown in Fig. 1 are plane parallel, we must have,

$$\mathbf{n_1} = -\mathbf{n_2} = \mathbf{n_3} = -\mathbf{n_4} = \mathbf{n_5}$$

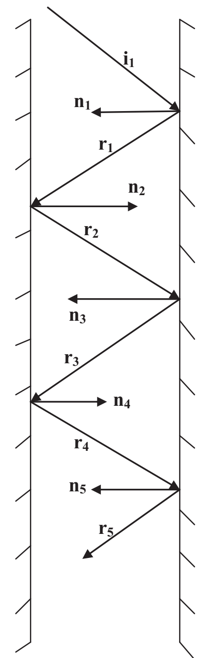

Fig. 1. Diagram showing multiple reflection in two plane parallel mirrors.

Hence the above Eqs. (1)–(5) will be respectively reduced to

$$\mathbf{n_1} \times \mathbf{i_1} = \mathbf{n_1} \times \mathbf{r_1}$$

$$\mathbf{n_1} \times \mathbf{r_1} = \mathbf{n_1} \times \mathbf{r_2}$$

$$\mathbf{n_1} \times \mathbf{r_2} = \mathbf{n_1} \times \mathbf{r_3}$$

$$\mathbf{n_1} \times \mathbf{r_3} = \mathbf{n_1} \times \mathbf{r_4}$$

and

$$\mathbf{n_1} \times \mathbf{r_4} = \mathbf{n_1} \times \mathbf{r_5} = \mathbf{n_5} \times \mathbf{r_5} \quad [\text{Since here,} \quad \mathbf{n_1} = \mathbf{n_5}]$$

It then follows from the above set of five relations that, in case of five successive reflection in two plane parallel mirrors, the generalized vectorial law of five such multiple reflection will assume the form,

$$\mathbf{n_1} \times \mathbf{i_1} = +(\mathbf{n_5} \times \mathbf{r_5})$$

It may be noted that in the aforesaid five successive reflection, there exists even number (four number) of rays (represented by the unit vectors $\mathbf{r_1}$, $\mathbf{r_2}$, $\mathbf{r_3}$, and $\mathbf{r_4}$), for each of which the two adjacent normal lie on opposite sides of such a ray. That is why the +ve sign appears on the right hand side of the above relation.

If, however, we consider four successive reflection in Fig. 1 starting from the first one, there exists odd number (three number) of rays, those represented by the unit vectors $\mathbf{r_1}$, $\mathbf{r_2}$, and $\mathbf{r_3}$ in Fig. 1, for each of which the two adjacent normal lie on opposite sides of such a ray. Then the above relation would have been reduced to the form,

$$\mathbf{n_1} \times \mathbf{i_1} = -(\mathbf{n_4} \times \mathbf{r_4})$$

Thus in general, the generalized vectorial law of 'p' number of successive reflection in two plane parallel mirrors ($p > 1$) must take up the form,

$$\mathbf{n_1} \times \mathbf{i_1} = \pm(\mathbf{n_p} \times \mathbf{r_p}),$$

where $\mathbf{i_1}$ = unit vector along the direction of the initial incident ray, $\mathbf{r_p}$ = unit vector along the direction of the reflected ray corresponding to the pth reflection, $\mathbf{n_1}$ = unit vector along the direction of the positive unit normal to the reflector corresponding to the first