Optik 125 (2014) 280–284

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Effects of atmospheric turbulence on the mode weight of the Laguerre-Gaussian Schell beams

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ARTICLE INFO

Article history:
Received 9 February 2013
Accepted 20 June 2013

Keywords:
Laguerre-Gaussian Schell beam
Mode weight
Orbital angular momentum
Mode space spread

ABSTRACT

Based on the Rytov approximation, we analyze the effects of the turbulence on the crosstalk and spatial spread of orbital angular momentum modes of the Laguerre-Gaussian Schell (LGS) beams. Our results show that the spatial incoherence of light source increases the crosstalk effect of atmospheric turbulence among the orbital angular momentum states and the spatial spread of modes. The mode weight decays with the mode quality factor increasing and the refractive index structure parameter increasing. The quality factor of LGS modes increases with the increasing of refractive index structure parameter in approximate linear form.

1. Introduction

The orbital angular momentum (OAM) of light has been attracting increasing interest in the past years, owing its potential for multivalue encoding of information [1]. However, the spatial-structure nature of OAM implies that it may be susceptible to atmospheric turbulence. Several models have recently been established to use the OAM states of light as a basic set for impressing coding information onto light field propagation in turbulence atmosphere [2–10], and the measuring method of the OAM spectrum of light from an extended natural source has been proposed [11–13]. The Kolmogorov atmospheric turbulence aberrations cause the crosstalk among the OAM states of single photons, reduce information capacity of the communication channel [2,3]. In the case of non-Kolmogorov turbulence, the crosstalk among orbits increases as the non-Kolmogorov parameter increases [4] and the turbulence also induces attenuation and crosstalk among multichannel free-space optical communication channels [5,6], the degradation in mode quality results in crosstalk between OAM modes [7]. The effects of atmospheric turbulence tilt, defocus, astigmatism, coma and Z-tilt corrected residual aberrations on the orbital angular momentum measurement probability of photons propagating in Kolmogorov/non-Kolmogorov turbulence channel are different [8–10]. It is shown that partially coherent beams are less sensitive to the effects of turbulence in contrast to fully coherent ones [14]. But, as we know, there are almost no discussions with respect to the effects of turbulence on the crosstalk weight of the OAM modes of Laguerre-Gaussian Schell (LGS) beams in view of the turbulence spread of the beam.

In this paper we model the effects of turbulence on the mode weight of the orbital angular momentum existing turbulence spread of the beam for LGS beams through the atmosphere.

2. Mode weight

The light field distribution of LGS beams at the source plane ($z=0$) can be described, in cylindrical coordinates, as

$$LG_0(r,\varphi) = LG_{l_0,p_0}(r,\varphi) f_s(r,\varphi) \tag{1}$$

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http://dx.doi.org/10.1016/j.ijleo.2013.06.048

where $r = |\mathbf{r}|$, $\mathbf{r} = (x, y)$ is the two-dimensional position vector in the source plane and φ is the azimuthally angle, LG_{l_0,p_0} is the normalized LG model at its beam waist ($z = 0$) with the radial mode order p_0 and azimuthal mode order l_0. $f_s(r, \varphi)$ is the correlation factor of the Schell source [15]

$$\mu(r, r', \varphi, \varphi') = f_s(r, \varphi) f_s^*(r, \varphi) = \exp\left[-\frac{r'^2 + r^2 - 2r'r\cos(\varphi' - \varphi)}{\rho_{s0}^2}\right], \quad \rho_{s0} > 0 \tag{2}$$

where ρ_{s0}^2 is the spatial correlation length of the source at plane $z = 0$.

In the weak atmospheric turbulence region and at any point in the half-space $z > 0$, the complex amplitude of LGS beam is given by

$$LG(r, \varphi, z) = LG_{l_0,p_0}(r, \varphi, z) f_s(r, \varphi) \exp[\psi(r, \varphi, z)] \tag{3}$$

where $\psi(r, \varphi, z)$ is complex phase of spherical waves propagating through turbulence and $LG_{l_0,p_0}(r, \varphi, z)$ is the normalized LG model at the z plane. In the paraxial approximation, $LG_{l_0,p_0}(r, \varphi, z)$ has the form [16]

$$LG_{l_0,p_0}(r, \varphi, z) = \frac{1}{\sqrt{2\pi}} R_{l_0,p_0}(r, z) \exp[il_0\varphi - i(2p_0 + |l_0| + 1)\delta] \tag{4}$$

where l_0 corresponds to the orbital angular momentum, $l_0\hbar$, carried by the beam and describes the helical structure of the wave front around a wave front singularity, p_0 is the number of radial zero crossings and δ is the Gouy phase. The function $R_{l_0,p_0}(r, z)$ is given by

$$R_{l_0,p_0}(r, z) = \frac{1}{w}\sqrt{\frac{2p_0!}{(p_0 + |l_0|)!}} \left(\frac{r}{w}\right)^{|l_0|} L_{p_0}^{|l_0|}\left(\frac{r^2}{w^2}\right) \exp\left(-\frac{r^2}{2w^2}\right) \exp\left(-\frac{ikr^2}{4R}\right) \tag{5}$$

where $L_{p_0}^{|l_0|}$ is an associated Laguerre polynomial, $w = w_0\sqrt{1 + (z/z_R)^2}$ is the spot size, w_0 is the beam waist at the plane $z = 0$, $z_R = 1/2kw_0^2$ is the Raleigh range and $R = z[1 + (z_R/z)^2]$ is the radius of wavefront curvature.

In the paraxial region, LG modes $LG_{l,p}(r, \varphi, z)$ form a complete set of orthonormal modes. So we expand $LG(r, \varphi, z)$ into the LG mode $LG_{l,p}(r, \varphi, z)$ and the $LG(r, \varphi, z)$ is written as

$$LG(r, \varphi, z) = \sum_{l,p} a_{l,p}(z) LG_{l,p}(r, \varphi, z) \tag{6}$$

The mode amplitudes $a_{l,p}(z)$ are given by the overlap integral

$$a_{l,p}(z) = \iint R_{l,p}^*(r) \frac{\exp(-i|l|\varphi)}{\sqrt{2\pi}} \exp[i(2p + |l| + 1)\delta] LG(r, \varphi, z) r dr d\varphi \tag{7}$$

where * denotes complex conjugate.

With the help of Eq. (6), the mode weight of finding on photon in the signal mode $\left|l, p\right\rangle$ can be written as

$$|a_{l,p}|^2 = \iint \iint R_{l,p}^*(r, \varphi, z) R_{l,p}(r', \varphi', z) \frac{\exp[-il(\varphi - \varphi')]}{2\pi} \left\langle LG^*(r', \varphi', z) LG(r, \varphi, z)\right\rangle r' r dr' d\varphi' dr d\varphi \tag{8}$$

where $\langle \bullet \rangle$ represents an ensemble average of atmospheric turbulence and $\left\langle LG^*(r', \varphi', z) LG(r, \varphi, z)\right\rangle$ is given by [4]

$$\left\langle LG^*(r', \varphi', z) LG(r, \varphi, z)\right\rangle = \frac{\mu(r, r')}{2\pi} R_{l_0,p_0}(r, \varphi) R_{l_0,p_0}^*(r', \varphi') e^{[il_0(\varphi - \varphi')]} \exp\left[-\frac{r'^2 + r^2 - 2r'r\cos(\varphi' - \varphi)}{\rho_0^2}\right] \tag{9}$$

Here $\rho_0 = \left\{[2\Gamma(7/6)\Gamma(11/6)]/\pi^{3/2} k^2 \sin(11\pi/6) \int_0^z C_n^2(\xi)(1 - \xi/z)^{5/3} d\xi\right\}^{-3/5}$ is the spatial coherence radius of a spherical wave propagating in the Kolmogorov turbulence, $C_n^2(z)$ is the refractive index structure parameter in a slat turbulence channel and in the Hufnagel-Velly model [17]. Based on the integral expression [18]

$$\int_0^{2\pi} \exp[-in\varphi_1 + \eta\cos(\varphi_1 - \varphi_2)] d\varphi_1 = 2\pi \exp(-in\varphi_2) I_n(\eta) \tag{10}$$

where $I_n(\eta)$ is the Bessel function of second kind with n order, and

$$\int_0^{2\pi} \exp\left[-i(l - |l_0|)\varphi_1 + \left(\frac{1}{\rho_0^2} + \frac{1}{\rho_s^2}\right) 2r'r\cos(\varphi_1 - \varphi_2)\right] d\varphi_1 = 2\pi \exp[-i(l - |l_0|)\varphi_2] I_{l-|l_0|}\left[2r'r\left(\frac{1}{\rho_0^2} + \frac{1}{\rho_s^2}\right)\right], \tag{11}$$

we have the mode weight of funding one photon in the signal mode $\left|l, p\right\rangle$

$$|a_{l,p}(z)|^2 = \iint R_{l,p}^*(r, z) R_{l,p}(r', z) R_{l_0,p_0}(r, z) R_{l_0,p_0}^*(r', z) \exp\left[\frac{-(r'^2 + r^2)}{\tilde{\rho}_0^2}\right] I_{l-|l_0|}\left(\frac{2rr'}{\tilde{\rho}_0^2}\right) rr' dr dr' \tag{12}$$

where $\tilde{\rho}_0^{-2} = (\alpha_s^0 \rho_0)^{-2}$ is the effective coherence length of LGS beams in atmospheric turbulence with $(\alpha_s^0)^2 = (1 + \rho_0^2/\rho_s^2)^{-1}$, which include the effects of spatial coherence of the source and the correlation length of phase fluctuations. From the definition of the effective coherence length $\tilde{\rho}_0$, we have a conclusion that the spatial coherence of the source only is a adjust factor for the spatial coherence length ρ_0 of atmospheric turbulence and as the increase of spatial coherence of the source the effective coherence length $\tilde{\rho}_0$ is also increasing. It is say that the spatial incoherence of the source will increase the effect of atmospheric turbulence on LG modes.