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Adaptive synchronization for dynamical networks of neutral type with time-delay

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ABSTRACT

This paper investigates adaptive synchronization for dynamical networks of neutral type with time-delay. In comparison with those of the existing synchronization of dynamical networks of neutral type with time-delay, we assume that the given neutral type expression can be linear function, nonlinear function, or even any elementary transformation. Based on the Lyapunov stability theorem, the adaptive control law is derived to make the state of two dynamical networks of neutral type synchronized. Some numerical are also given to show the effectiveness of the proposed method.

1. Introduction

In the past few decades, the problem of control and synchronization about complex dynamical networks has been extensively investigated in various fields of science and engineering due to its many potential practical applications [1–11]. Recently, the practicality of neutral-type models attracts researchers to investigate the stability and stabilization of the neutral-type neural networks [12–16]. Due to the fact that many systems in the real world can be described by neutral-type neural networks, the investigation on the synchronization of coupled neutral-type neural networks have a lot of potential applications in many areas [17–22]. However, the synchronization for dynamical networks of neutral type has been rarely researched. In this paper, we consider the dynamical networks model of neutral type, the given neutral type expression can be linear function, nonlinear function, or even any elementary transformation, such neutral type model is a more general, the proposed neutral type expression encompasses the model discussed in Ref. [23], and such model has been ignored by the existing synchronization schemes in the literatures. Motivated by the above discussions, our main target is to find sufficient conditions to ensure the adaptive synchronization for dynamical networks of neutral type with time-delay.

This work is organized as follows. Section 2 describes the model and the preliminaries. Section 3 investigates adaptive synchronization for dynamical networks of neutral type with time-delay. Section 4 presents an example and the related simulation results. Section 5 gives the conclusion of the paper.

2. Model and preliminaries

The dynamical behavior of the neural network we consider is assumed to be governed by the following system of ordinary differential equations:

$$d(x^i(t)+g(x^i(t-\tau))) = \left[f(x^i(t)) + \sum_{j=1}^{N} a_{ij}x^j(t) + \sum_{j=1}^{N} b_{ij}x^j(t-\tau) \right] dt, \tag{1}$$

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where $x^i = (x_1^i, x_2^i, \ldots, x_n^i)^T \in R^n$ is the state vector of node i, f: $R^n \to R^n$ is a vector value function standing for the activity of an individual subsystem. $A = (a_{ij})_{N\times N} \in R^{N\times N}$ and $B = (b_{ij})_{N\times N} \in R^{N\times N}$ are the coupling matrixes, a_{ij} and b_{ij} are the weight or coupling strength. If there exists a link from node i to j $(i \neq j)$ then $a_{ij} \neq 0$ and $b_{ij} \neq 0$. Otherwise, $a_{ij} \neq 0$ and $b_{ij} \neq 0$, and $a_{ij} = a_{ij}$, $b_{ij} = b_{ij}$, $a_{ii} = -\sum_{j=1, i\neq j}^{N} a_{ij}$, $b_{ii} = -\sum_{j=1, i\neq j}^{N} b_{ij}$, $i = 1, 2, \ldots, N$.

Remark 1. Here, $g(\cdot)$ can be linear function, nonlinear function, or even any elementary transformation, such neutral type model is a more general, can be applied to some delayed neutral network with neutral type.

In the paper, we have the following mathematical preliminaries.

Definition 1. The array of systems in the complex networks is said to be synchronization if $\lim_{t\to\infty}||y^i(t) - x^i(t)||=0$ for any $i = 1, 2, \ldots, N$.

Assumption 1. We also assume that f is Lipschitz with respect to its argument i.e.

$$|f(y^i(t)) - f(x^i(t))| \leq \eta e^i(t),\ \eta > 0,$$

where $e^i(t) = y^i(t) - x^i(t)$.

Lemma 1 ([24]). For any vectors $x, y \in R^m$ and positive definite matrix $Q \in R^{m\times m}$, the following matrix inequality holds: $2x^T y \leq x^T Qx + y^T Q^{-1} y$. If not specified otherwise, inequality $Q > 0$ ($Q < 0$, $Q \geq 0$, $Q \leq 0$) means Q is a positive (or negative, or semi-positive, or semi-negative) definite matrix.

Lemma 2 ([25]). (The Shur complements) Let Q and R be two symmetric matrices. Then $\begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} < 0$ is equivalent to that $R < 0$ and $Q - SR^{-1}S^T < 0$.

3. Synchronization criteria

We take the network given by Eq. (1) as the driving network, and a response network is given by

$$d(y^i(t) + g(y^i(t-\tau))) = \left[f(y^i(t)) + \sum_{j=1}^{N} a_{ij} y^j(t) + \sum_{j=1}^{N} b_{ij} y^j(t-\tau) + u^i \right] dt. \quad (2)$$

Theorem 1. Under Assumption 1, the controlled complex dynamical network (2) synchronizes to the complex dynamical network (1) using the following controller:

$$u^i = -\alpha^i(t)(e^i(t) + g(e^i(t-\tau))) \quad (3)$$

$$\dot{\alpha}^i(t) = \theta^i (e^i(t) + g(e^i(t-\tau)))^T (e^i(t) + g(e^i(t-\tau))), \quad (4)$$

where θ_i is positive constant.

Proof. Let $e^i(t) - y^i(t) - x(t)$, then we have the following dynamical error equations:

$$d(e^i(t) + g(e^i(t-\tau))) = \left[f(y^i(t)) - f(x^i(t)) + \sum_{j=1}^{N} a_{ij} e^j(t) + \sum_{j=1}^{N} b_{ij} e^j(t-\tau) + u^i \right] dt. \quad (5)$$

We choose a non-negative function as

$$V(t) = \frac{1}{2}\sum_{i=1}^{N} [e^i(t) + g(e^i(t-\tau))]^T [e^i(t) + g(e^i(t-\tau))] + \frac{1}{2}\sum_{i=1}^{N} \frac{1}{\theta^i} (\alpha^i(t) - k)^2 + \sum_{j=1}^{N} \int_{t-\tau}^{t} (e^i(s))^T e^i(s) ds.$$