



# An analytical study of a doubly clad compressed ellipse optical waveguide



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## ARTICLE INFO

### Article history:

Received 18 February 2013

Accepted 30 June 2013

### Keywords:

Doubly clad optical waveguides

Compressed ellipse

Cutoff conditions, Characteristic equation

Dispersion curves

## ABSTRACT

Using an analytical method based on boundary matching technique, the modal behavior and cutoff frequencies of a compressed ellipse doubly clad optical waveguide is studied. The proposed waveguide consists of a core region of higher refractive index with two cladding regions: one is inner cladding and the other is outer cladding. We take appropriate orthogonal coordinates for the proposed structure and impose the boundary conditions to obtain the characteristic equation. The effect of the width of inner cladding layer on the dispersion characteristic is observed. It is found that the width of inner cladding is able to tailor the dispersion characteristic and cutoff condition of the waveguide up to a certain limit.

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## 1. Introduction

It is a well known fact that the optical fiber is the back bone of modern communication systems. Basic optical fiber consists of a cylindrical core of high refractive index surrounded by a cylindrical cladding of slightly low refractive index in which light propagate through fiber by the principle of total internal reflection. The conventional optical fiber is made by silica to provide very low attenuation of the signal. Nowadays these waveguides are widely used in the integrated optics, optical waveguide sensors, optical switches, etc. where we need to modify its propagation characteristics. There are two approaches through which one can tailor the propagation characteristics of these waveguides: one is changing the cross sectional view of a waveguide [1–5] and the other is changing the constituent materials of core or cladding regions [6–9]. However, the doubly clad waveguide [10,11] is a class of optical waveguide where one can tailor the propagation characteristics of waveguides up to a certain limit by modifying the cladding parameters [12].

Since it is difficult to make a perfectly circular fiber waveguide, the elliptical waveguides have drawn the attention of many researchers [13–17] due to their closeness to the practical optical fiber waveguide. Also, there are various techniques through which one can analyze an elliptical waveguide [18,19]. Using a very simple

approach i.e. by matching the electric fields and its derivatives on the interfaces, Singh et al. [20,21] analyzed the unconventional optical waveguide having large number of cladding.

Keeping the above discussion in mind, we choose here a deformed doubly clad elliptical optical waveguide and study the effect of deformation on the dispersion curves. This deformation is deliberately introduced in the elliptical waveguide by compressing it along its minor axis. Since our basic aim is to obtain extra degree of freedom to control the dispersion characteristic by introducing the inner cladding region therefore, we are not considering the birefringence effect of the proposed structure. To outline the present article, dispersion relation of the proposed structure is given in Section 2. In Section 3 numerical results and discussions are presented. Finally, the paper ends with some remarks in the conclusion given in Section 4.

## 2. Theory

Fig. 1(a) shows the cross sectional view of the proposed waveguide. The shape of the cross-section is that of an ellipse compressed along its minor axis (ECMI). The waveguide consists of a core of refractive index  $n_1$ , inner cladding of refractive index  $n_2$  and outer cladding of refractive index  $n_3$  as shown in Fig. 1(b). The proposed structure is mathematically represented as

$$r = ae^{(1/2)\sin p\theta} \quad (1)$$

where  $a$  is the size parameter and we choose  $p=2$  for the ECMI waveguide.

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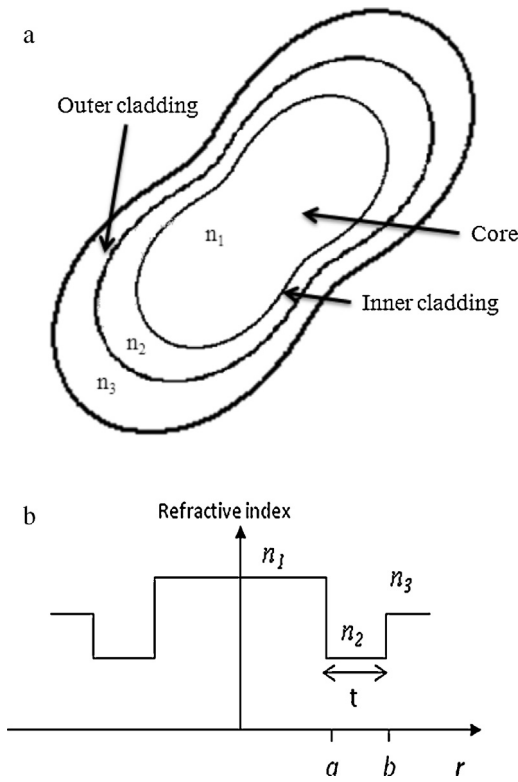


Fig. 1. (a) The cross-sectional view of a compressed ellipse doubly clad optical waveguide. (b) Variation of refractive index profile with core diameter.

This structure of the waveguide can easily be solved by choosing an appropriate coordinate system. To obtain such coordinates, one uses the points of intersection of the curve represented in Eq. (1) with its normal curves. The curve normal to Eq. (1) is

$$r = c [\cos^2 p\theta(1 + \sin(p\theta))^{-2}]^{(1/p^2)} \tag{2}$$

where  $c$  is another size parameter. The new coordinates of the point  $(r, \theta)$  are thus  $a, c$ . Using Eqs. (1) and (2) with some straightforward steps one can obtain the scale factors  $h_1, h_2$ , and  $h_3$ , which are given as

$$h_1 = \frac{pr \left[ 1 - \left( \frac{2p^2}{p^2 + 4} \right) \ln(c/a) \right]^{(1/2)}}{c \left[ p^2 \left( 1 - \left( \frac{2p^2}{p^2 + 4} \right) \ln(c/a) \right)^2 + 4 \right]}$$

$$h_2 = \frac{2r}{a \left[ p^2 \left( 1 - \left( \frac{2p^2}{p^2 + 4} \right) \ln(c/a) \right)^2 + 4 \right]}$$

$$h_3 = 1$$

Here, we observed that the third coordinate is left unaltered so that, instead of the coordinates  $(r, \theta, z)$ , we now have new coordinates  $(a, c, z)$ . Now, in general the scalar wave equation is given as

$$\nabla^2 E_z + \omega^2 \mu \epsilon E_z = 0 \tag{3}$$

where  $\nabla^2 = \frac{1}{h_1 h_2 h_3} \left[ \frac{\partial}{\partial a} \left( \frac{h_2 h_3}{h_1} \frac{\partial}{\partial a} \right) + \frac{\partial}{\partial c} \left( \frac{h_1 h_3}{h_2} \frac{\partial}{\partial c} \right) + \frac{\partial}{\partial z} \left( \frac{h_1 h_2}{h_3} \frac{\partial}{\partial z} \right) \right]$  and  $E_z$  is the  $z$ -component of the electric field. To proceed further with this differential equation would be a difficult task unless some simplification assumption is made. Therefore, we assume  $a \approx c$ , it may be a possible special case which refers to the compressed part of the ECMI shape. Hence using this approximation, we obtain

$$\left[ \frac{1}{a^2} \frac{\partial^2 E_z}{\partial a^2} - \frac{1}{a^3} \frac{\partial E_z}{\partial a} + \frac{1}{c^2} \frac{\partial E_z}{\partial c^2} - \frac{1}{c^3} \frac{\partial E_z}{\partial c} \frac{1}{2ac} \frac{\partial^2 E_z}{\partial z^2} \right] + \frac{1}{2ac} \omega^2 \mu \epsilon E_z = 0 \tag{4}$$

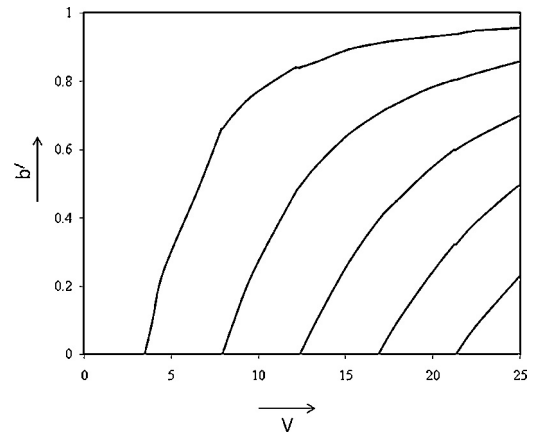


Fig. 2. Dispersion curves for the proposed waveguide having inner cladding width  $t = 0.01 \mu\text{m}$ .

where  $\omega$  is the angular frequency,  $\epsilon$  is the permittivity of the dielectric media,  $\mu_0$  is the permeability.

Using the technique of separation of variables we obtain two equations, each of which is in one variable.

$$E_z = E_1(a)E_2(c) \exp \{j(\omega t - \beta z)\} = 0 \tag{5}$$

$$\frac{\partial^2 E_2(c)}{\partial c^2} - \frac{1}{c} \frac{\partial E_2(c)}{\partial c} = -\nu \tag{6}$$

$$\frac{\partial^2 E_1(a)}{\partial a^2} - \frac{1}{a} \frac{\partial E_1(a)}{\partial a} + \frac{1}{2}(\omega^2 \mu \epsilon_1 - \beta^2)E_1(a) = \nu \tag{7}$$

where  $\beta$  is the  $z$ -component of propagation constant and  $\nu$  is the mode designating parameter and for the lowest order it is taken as  $\nu = 0$ . Since Eq. (6) has no propagation parameters, we concentrate only on Eq. (7). Eq. (7) for the core region is written as

$$\frac{\partial^2 E_1(a)}{\partial a^2} - \frac{1}{a} \frac{\partial E_1(a)}{\partial a} + \frac{1}{2}U^2 E_1(a) = 0 \tag{8a}$$

and for the cladding regions it is written as

$$\frac{\partial^2 E_1(a)}{\partial a^2} - \frac{1}{a} \frac{\partial E_1(a)}{\partial a} - \frac{1}{2}W_i^2 E_1(a) = 0 \tag{8b}$$

where  $U^2 = \omega^2 \mu \epsilon_1 - \beta^2$  and  $W_i^2 = \beta^2 - \omega^2 \mu \epsilon_i$ ,  $\epsilon_1$  and  $\epsilon_i$  being the permittivity of the core and cladding regions respectively with  $i = 2$  for inner cladding and  $i = 3$  for outer cladding regions.

The solution of Eq. (8) in core region is written as

$$E_1(a) = a \left[ A J_1 \left( \frac{1}{\sqrt{2}} U a \right) + B Y_1 \left( \frac{1}{\sqrt{2}} U a \right) \right] \text{ (core region)} \tag{9}$$

and the solution of Eq. (8) in cladding regions are written as

$$E_1(a) = a \left[ C I_1 \left( \frac{1}{\sqrt{2}} W_2 a \right) + D K_1 \left( \frac{1}{\sqrt{2}} W_2 a \right) \right] \text{ (inner cladding)} \tag{10a}$$

$$E_1(a) = a \left[ G K_1 \left( \frac{1}{\sqrt{2}} W_3 a \right) \right] \text{ (outer cladding)} \tag{10b}$$

The boundary conditions of the proposed waveguide are given as

$$E_{1\text{core}}|_a = E_{1\text{inner clad}}|_a$$

$$E'_{1\text{core}}|_a = E'_{1\text{inner clad}}|_a$$

$$E_{1\text{inner clad}}|_{a+t} = E_{1\text{outer clad}}|_{a+t}$$

$$E'_{1\text{inner clad}}|_{a+t} = E'_{1\text{outer clad}}|_{a+t}$$

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