Optik 124 (2013) 2686–2690

Contents lists available at SciVerse ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Interaction between parallel Gaussian electromagnetic beams in a plasma with weakly relativistic-ponderomotive regime

R. Mahajan*, T.S. Gill, R. Kaur

Department of Physics, Guru Nanak Dev University, Amritsar 143005, India

ARTICLE INFO

Article history:
Received 24 April 2012
Accepted 2 August 2012

Keywords:
Gaussian beam
Self-focusing
Relativistic
Ponderomotive

ABSTRACT

In this communication, authors have studied the interaction between two Gaussian electromagnetic beams in a plasma for weakly relativistic-ponderomotive regime when the axes of two beams are initially parallel along *z*-axis in the *xz* plane: the beams are initially propagating in the *z*-direction. Second order coupled ordinary differential equations have been obtained for the distance between centers of the beams and the beam widths in the *x*- and *y*-directions along *z*-axis. The so obtained equations are solved numerically for a range of parameters and the results are depicted in the form of graphs.

1. Introduction

Theoretical and experimental study of evolution of high intensity laser beams as they propagate through plasmas is an active area of research due to its importance in potential applications such as plasma based accelerators [1], inertial confinement fusion [2,3], ionospheric modification [4,5] and new radiation sources [6,7]. For success of these applications, a long propagation distance of intense lasers in plasma is desirable. As several effects emerge in such transit of high power laser beam through plasma medium, many instabilities and nonlinear phenomena such as self-phase modulation (SPM), the filamentation instability, group velocity dispersion (GVD), finite pulse effects, relativistic and ponderomotive self-focusing effects, become important. Therefore, it is important to study analytically and numerically some of these effects. This leads to a deep understanding of laser–plasma-interaction physics to keep the nonlinear process at low level.

Ever since the first investigation reported by [8] and subsequently followed by [9] on self-trapping of optical beams, this nonlinear generic process had been focus of attention for nearly five decades and is still being actively pursued by researchers worldwide because of their relevance to a number of newly discovered process. The self-focusing being a genuinely nonlinear phenomenon arising out of nonlinear response of material medium leading to its modified refractive index. Specifically, in the laser–plasma-interaction, the generic process of self-focusing of the laser beams [9–18,20,19,21,22] has been focus of attention as it affects many other nonlinear processes. It plays crucial role in beam propagation and arises due to increase of the on-axis index of refraction relative to edge of the laser beam. For example, for ponderomotive force type nonlinearity, electrons are expelled from the region of high intensity laser field. On the other hand, relativistic self-focusing results from the effect of quiver motion leading to reduced local plasma frequency. The self-focusing is counterbalanced by the tendency of the beam to spread because of diffraction. In the absence of nonlinearities, the beam will spread substantially in a Rayleigh length, R_d ($\sim ka_0^2$), where k is the wavenumber and a_0 is the spot size of the laser beam.

Theoretical analysis of self-focusing, self-trapping and filamentation of laser has been reported by Akhmanov et al. [23], developed by Sodha et al. [12] and Kaw et al. [24]. Sodha et al. extended the laser beam self-focusing to variety of media such as dielectrics, semiconductors and plasmas under considering different nonlinear mechanisms [11]. Self-focusing and filamentation are among the most dangerous nonlinear phenomena which destroy the uniformity of overall irradiation required for direct-drive fusion experiment as well as leads to seeding and growth of hydrodynamic instabilities. Experimental as well as theoretical observations of relativistic self-focusing and ponderomotive self-channeling have been reported in a number of investigations [25–32]. The dynamics of ponderomotive channeling in underdense plasma has recently been studied experimentally [33]. Relativistic laser–plasma-interaction physics has also been focus of attention as many nonlinear processes playing key role in the generation of new ion sources as reported recently [34,35]. Further, there have been a series of novel experiments to study rich physics issues in nuclear and particle physics, atomic physics [36], plasma physics [37,38] and applied sciences [39,40].

* Corresponding author. Tel.: +91 08146292032.
E-mail address: ranju.mhjn@gmail.com (R. Mahajan).

0030-4026/$ – see front matter © 2012 Elsevier GmbH. All rights reserved.
http://dx.doi.org/10.1016/j.ijleo.2012.08.095

The interaction between electromagnetic beams has been investigated recently [41–43]. Ren et al. [41] used the variational method as well as a three dimensional particle-in-cell model of laser light in a plasma to demonstrate the mutual attraction between two electromagnetic beams. Shukla et al. [44] have recently studied the nonlinear interaction between two weakly relativistic crossing laser beams in plasmas. Sodha et al. [45] have studied the basic physics of interaction for identical incoherent beams. Dielectric function has a maximum or minimum in the space between two beams depending on whether $2x_0 < \omega r_0$ or $2x_0 > \omega r_0$. Two beams attract each other when the condition $2x_0 < \omega r_0$ is satisfied and repel each other for $2x_0 > \omega r_0$. Sodha et al. [46] have similarly obtained the space-dependent distribution of the electron temperature and thereby the dielectric function for the case of self-focusing of single beam in a collisional plasma with thermal conduction.

In the present investigation, authors have studied the interaction between two Gaussian electromagnetic beams in a plasma for weakly relativistic-ponderomotive regime when the axes of two beams are initially parallel along z-axis in the xz plane: the beams are initially propagating in the z-direction. Second order coupled ordinary differential equations have been obtained for the distance between centers of the beams and the beam widths in the x- and y-directions along z-axis. The so obtained equations are solved numerically for a range of parameters and results are also discussed in detail.

2. Basic formulation

The intensity distribution of two Gaussian beams propagating along the z-axis at $z = 0$ is given by:

$$E_1E_1^* = E_{10}^2 exp\left[\frac{(x+x_0)^2}{r_1^2} - \frac{y^2}{r_1^2}\right] \tag{1}$$

and

$$E_2E_2^* = E_{20}^2 exp\left[\frac{(x-x_0)^2}{r_2^2} - \frac{y^2}{r_2^2}\right] \tag{2}$$

where E_1 and E_2 are the complex amplitudes of the electric vectors of two beams. $2x_0$ is the distance between the axes of beams at $z = 0$ and r_1 and r_2 are the widths of two beams at $z = 0$. The nonlinear dielectric function ϵ_j in an isotropic inhomogeneous medium can be expressed as:

$$\epsilon_j = \epsilon_j(z, E_1E_1^*, E_2E_2^*) \tag{3}$$

where E_j is the amplitude of electric vector of the jth beam ($j = 1$, 2). One can expand ϵ_j in powers of x and y and in the paraxial ray approximation retain terms only up to those having x^2 and y^2. Thus, one obtains:

$$\begin{aligned}\epsilon_j &= \epsilon_{0j}(z, E_{10}E_{10}^*, E_{20}E_{20}^*) - x\epsilon_{1j}(z, E_{10}E_{10}^*, E_{20}E_{20}^*)\\ &\quad - x^2\epsilon_{2xj}(z, E_{10}E_{10}^*, E_{20}E_{20}^*) - y^2\epsilon_{2yj}(z, E_{10}E_{10}^*, E_{20}E_{20}^*)\end{aligned} \tag{4}$$

In the steady state, amplitude of the electric vector E_j satisfies the wave equation:

$$\nabla^2 E_j + \frac{\omega_j^2}{c^2}\epsilon_j E_j = 0 \tag{5}$$

Eq. (5) has a solution of the form:

$$E_j(x, y, z) = A_j(x, y, z) exp\left[-\iota\int_0^z k_j(z)dz\right] \tag{6}$$

where $A_j(x, y, z)$ is the complex amplitude of electric field and

$$k_j^2 = \frac{\omega_j^2}{c^2}\epsilon_{0j} \tag{7}$$

Substituting for $E_j(x, y, z)$ from Eq. (6) in Eq. (5) and neglecting $(\partial^2 A_j)/(\partial z^2)$ one obtains:

$$\frac{\partial^2}{\partial x^2}A_j + \frac{\partial^2}{\partial y^2}A_j - 2\iota k_j\frac{\partial A_j}{\partial z} - \iota A_j\frac{\partial k_j}{\partial z} - k_j^2A_j + \frac{\omega_j^2}{c^2}\epsilon_j(x, y, z)A_j = 0 \tag{8}$$

For a nearly spherical wave front

$$A_j(x, y, z) = A_{j0}(x, y, z) exp[-\iota k_j(z)S_j(x, y, z)] \tag{9}$$

and the eikonal

$$S_j = \frac{x_j^2}{2}\beta_{1j}(z) + x_j\beta_{2j}(z) + \frac{y^2}{2}\beta_{3j}(z) + \Phi_j(z) \tag{10}$$

where

$$x_1 = x + x_0F_1 \quad \text{and} \quad x_2 = x - x_0F_2 \tag{11}$$

Substituting the values of $A_j(x, y, z)$ and S_j from Eqs. (9) and (10) and ϵ_j from Eq. (4) into Eq. (8) and then equating the real and imaginary parts on both sides of the resulting equation, we have:

$$\begin{aligned}2\frac{\partial S_j}{\partial z} &+ \left[\left(\frac{\partial S_j}{\partial x}\right)^2 + \left(\frac{\partial S_j}{\partial y}\right)^2\right] + \frac{2S_j}{k_j(z)}\frac{dk_j(z)}{dz}\\ &= \frac{1}{k_j^2A_{j0}}\left(\frac{\partial^2}{\partial x^2}A_{j0} + \frac{\partial^2}{\partial y^2}A_{j0}\right) - x\frac{\epsilon_{1j}(z)}{\epsilon_{0j}(z)}\\ &\quad - x^2\frac{\epsilon_{2xj}(z)}{\epsilon_{0j}(z)} - y^2\frac{\epsilon_{2yj}(z)}{\epsilon_{0j}(z)}\end{aligned} \tag{12}$$

$$\begin{aligned}\frac{\partial}{\partial z}A_{j0}^2 &+ A_{j0}^2\left(\frac{\partial^2}{\partial x^2}S_j + \frac{\partial^2}{\partial y^2}S_j\right) + \left(\frac{\partial A_{j0}^2}{\partial x}\frac{\partial S_j}{\partial x} + \frac{\partial A_{j0}^2}{\partial y}\frac{\partial S_j}{\partial y}\right)\\ &= \frac{A_{j0}^2}{k_j(z)}\frac{dk_j}{dz}\end{aligned} \tag{13}$$

Following [47] and after some algebraic manipulations, one obtains:

$$\begin{aligned}E_1E_1^* &= A_{10}^2(x, y, \xi)\\ &= \frac{E_{100}^2}{f_{1x}(\xi)f_{1y}(\xi)}\sqrt{\frac{\epsilon_{01}(0)}{\epsilon_{01}(\xi)}} exp\left(-\frac{(x + x_0F_1(\xi))^2}{r_1^2f_{1x}^2(\xi)} - \frac{y^2}{r_1^2f_{1y}^2(\xi)}\right)\end{aligned} \tag{14}$$

$$\begin{aligned}E_2E_2^* &= A_{20}^2(x, y, \xi)\\ &= \frac{E_{200}^2}{f_{2x}(\xi)f_{2y}(\xi)}\sqrt{\frac{\epsilon_{02}(0)}{\epsilon_{02}(\xi)}} exp\left(-\frac{(x + x_0F_2(\xi))^2}{r_2^2f_{2x}^2(\xi)} - \frac{y^2}{r_2^2f_{2y}^2(\xi)}\right)\end{aligned} \tag{15}$$

where $\xi = cz/(\omega_1 r_1^2)$ is the dimensionless distance of propagation. Substituting for A_{j0}^2 from Eqs. (14) and (15) and S_j from Eq. (10) into Eq. (12) and equating the coefficients of x, x^2 and y^2 on both sides for both the beams, we have:

$$\frac{d^2F_1}{d\xi^2} = -\frac{\rho_1^2}{\epsilon_{01}}\left[\epsilon''_{2x1} - \epsilon'_{11}\right] - \frac{1}{2\epsilon_{01}}\frac{d\epsilon_{01}}{d\xi}\frac{dF_1}{d\xi} \tag{16}$$

$$\frac{d^2f_{1x}}{d\xi^2} = -\frac{\rho_1^2}{f_{1x}\epsilon_{01}}\left[\epsilon'_{2x1} - \frac{1}{\rho_1^2f_{1x}^2}\right] - \frac{1}{2\epsilon_{01}}\frac{d\epsilon_{01}}{d\xi}\frac{df_{1x}}{d\xi} \tag{17}$$