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Bayesian sinogram smoothing with an anisotropic diffusion weighted prior for low-dose X-ray computed tomography

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ABSTRACT

As is known, low-dose computed tomography (CT) image can be severely degraded by the excessive quantum noise. In order to address this problem, we firstly present a novel anisotropic diffusion weighted prior applied in Bayesian-based statistical sinogram smoothing approach in this work. Then, the reconstructed image is obtained by the filtered back-projection (FBP) from the smoothed projection data. Compared with the traditional priors, the proposed novel prior can adaptively adjust the smoothing degree according to the sinogram characteristic. The effectiveness and feasibility of the proposed approach are validated by both digital phantom and clinical data experiments. The superiority of the presented method over other methods is also quantitatively studied by resolution–noise tradeoff curves and signal to noise ratio (SNR). The experimental results indicate that the developed approach has the excellent performance for low-dose CT imaging.

1. Introduction

Nowadays, owing to the excellent density resolution, X-ray computed tomography (CT) as an excellent medical imaging tool plays a very important role in the clinic diagnosis of many diseases. However, due to the relatively high X-ray dose exposure to patients and the increasing concern on the radiation risks, its further application is limited. Consequently, it is clinically desired to lower the X-ray radiation dose during the CT imaging, especially when screening or examining for pregnant woman and pediatric patient. The principle of "as low as reasonably achievable" (ALARA) was proposed as the goal for radiation dose in clinical practice [1].

Recently, many efforts have been made to control radiation exposure by improving the imaging technologies and avoiding unnecessary scans. Thereinto, a direct and effective measure to lower X-ray dose is to decrease the tube current (mA) in the CT data acquisition protocols. However, it is well known that the image quality would become worse as the tube current decreases, which is mainly because the measured data are contaminated by excessive quantum noise and there occurs the photon starvation phenomenon under this case. In general, this measurement noise could lead to an increase of overall noise level and visual streak artifacts in the reconstructed image. Up to now, there have been many algorithms to reduce the noise and improve the quality of reconstructed image for low-dose CT imaging. In the early stage, Hsieh [2] proposed a nonlinear filtering method, called as ATM, which can suppress the streak artifacts effectively. The later algorithms presented in [3–6] can be also regarded as the same class algorithms. Besides, statistical iterative reconstruction (SIR) algorithms in the image space have been also studied widely [7–9]. In the proposed strategies, another promising algorithm is to reconstruct the image by filtered back-projection (FBP) method from a Bayesian estimation of the noisy sinogram [10–12]. Due to explicitly considering the statistical property of measured data and incorporating the objective constraints and prior information into the reconstruction process, this algorithm can give rise to more accurate reconstructed result. In addition, because the iterative Bayesian optimal estimation is accomplished only in projection space, this algorithm also overcomes the disadvantages of time-consuming and bulky memory requirement owned by the SIR algorithms in image space.

The key point of Bayesian estimation is to select an effective prior about what the noise-free sinogram data should be like. Among the proposed priors, as an effective prior, Gaussian MRF prior is employed frequently [8,11–14]. For Bayesian reconstruction algorithm with MRF prior, whether the smoothness parameter controlling the smoothness degree of the desired sinogram is appropriately set is vital for the improvement of reconstructed image quality. In some literatures [11,12], a fixed smoothness parameter was employed for the whole sinogram. Due to the lack of consideration on the difference of noise characteristic in the

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different sinogram region, these methods are liable to lead to over-smoothing or excessive noisy effect [14].

In this paper, aiming at the above problem caused by fixed parameter problem, based on partial differential equation (PDE) theory, we firstly design an edge-preserving prior by introducing an anisotropic diffusion weighted coefficient to the fixed smoothness parameter prior for sinogram smoothing. Then, the low-dose X-ray CT image is reconstructed by FBP from the estimated projection data. This edge-preserving prior enables the Bayesian algorithm to automatically differentiate between edge and flat region and adaptively adjust the weight during the smoothing process. And the experimental results show the proposed algorithm can acquire a satisfactory trade-off between suppressing noise and preserving resolution of the reconstructed CT image.

2. Materials and methods

Under the Bayesian and MRF framework, regularization from prior information can be imposed on the statistical smoothing process of sinogram to suppress noise. And we can build the following posterior probability $P(g/y)$ in the sinogram space:

$$P\left(\frac{g}{y}\right) \propto P\left(\frac{g}{y}\right) P(g), \tag{1}$$

$$P(g) = Z^{-1} \exp(-\beta U(g)) = Z^{-1} \exp\left(-\beta \sum_j U_j(g)\right), \tag{2}$$

where vector y is the system-calibrated and log-transformed projection measurements and vector g is the ideal projection data to be estimated. $P(y/g)$ is the likelihood function, and $P(g)$ is the prior distribution. The partition function Z is a normalizing constant and is set to 1 in this study. $U_j(g)$ is the value of prior energy function $U(g)$ on the sinogram g at pixel j. Parameter β is the global hyperparameter controlling the degree of prior's influence on sinogram g. By taking the negative logarithm on (1), we can obtain the following posterior energy function:

$$\phi_\beta\left(\frac{g}{y}\right) = -L(y, g) + \beta \sum_j U_j(g), \tag{3}$$

where $L(y, g)$ represents the log-likelihood energy function. The smoothed sinogram can be obtained through minimization of $\phi_\beta(g/y)$, i.e.,

$$g = arg \min_{g \geq 0} \phi_\beta\left(\frac{g}{y}\right). \tag{4}$$

2.1. The traditional prior model

In (2), the different selection of energy function $U_j(g)$ leads to the different prior distribution model. Conventionally, an energy function which is easily analyzed and permits an optimal solution of (4) with less computational time has the following quadratic form

$$U_j(g) = \frac{1}{2} \sum_{i \in N_j} \omega_{i,j}(g_i - g_j)^2, \tag{5}$$

where N_j denotes the neighborhood centered on pixel j and the smoothing weight $\omega_{i,j}$ determines the interaction degree between pixel i and pixel j.

In this study, we employ (5) as the base of the proposed energy function and the reason is that the optimal solution of (4) can be obtained easily because this energy function is convex. Moreover, we select a four nearest neighborhood and set the smoothing weight $\omega_{i,j}$ for the two horizontal neighbors (along the bin direction) being quadruple of $\omega_{i,j}$ for the two vertical neighbors (along the angular direction) as described in [14] for 2-D sinogram smoothing, which considers that the correlation is stronger in the radial direction than in the angular direction. Specifically, the used neighborhood is shown in Fig. 1.

0	0.25	0
1	0	1
0	0.25	0

Fig. 1. The neighborhood and the smoothing weights.

2.2. The proposed anisotropic diffusion weighted prior

In the traditional fixed parameter energy function model (5), $\omega_{i,j}$ is invariable for the whole sinogram, which can result in over-smoothing or excessive noise effect owning to the same smoothing degree on the whole sinogram without considering the different characteristic of the different region. Therefore, it is very important to properly set this parameter in the sinogram smoothing process. For this reason, inspired by the PDE theory [15], we proposed an anisotropic diffusion weighted prior model by introducing an adaptive weighted coefficient $c(|\nabla g_j|, s_j^2)$ to (5), i.e.,

$$U_j(g) = \frac{1}{2} c(|\nabla g_j|, s_j^2) \cdot \sum_{i \in N_j} \omega_{i,j}(g_i - g_j)^2, \tag{6}$$

$$c(|\nabla g_j|, s_j^2) = \frac{1}{1 + (|\nabla g_j| \cdot s_j^2)^2 / t}, \tag{7}$$

where $|\nabla g_j|$ and s_j^2 is the modulus of gradient and the gray-level variance calculated from a 3×3 neighborhood of sinogram at pixel j, respectively. To be specific, the gradient modulus $|\nabla g_j|$ is simply calculated by the well-known Sobel mask. And the variance s_j^2 is given by

$$s_j^2(x, y) = \frac{1}{9} \sum_{i=-1}^{1} \sum_{j=-1}^{1} [I(x + i, y + j) - \bar{I}(x, y)]^2, \tag{8}$$

where $\bar{I}(x, y)$ is the mean value of gray values $I(x + i, y + j)$ in the 3×3 neighborhood window.

In the proposed model (6), the coefficient $c(\cdot)$ is a nonnegative monotonically decreasing function. The basic ideal of the proposed model is the penalty weight is able to vary with the product of $|\nabla g_j|$ and s_j^2. For those smaller values of the product of $|\nabla g_j|$ and s_j^2 corresponding to the relatively flat regions, the coefficient $c(\cdot)$ is approximate to one and the penalty weight is larger. Thus, the smoothing process is performed. However, for those larger values of the product of $|\nabla g_j|$ and s_j^2 corresponding to the edges or fine details the coefficient $c(\cdot)$ is approximate to zero and the penalty weight is smaller. As a result, the smoothing process is terminated while the edges are preserved well. Consequently, the smoothing parameter can adaptively vary according to the characteristic of the different sinogram region, which can result in a better estimation of sinogram. Obviously, in (7), t adjusting the amount of smoothing