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The measurement of Faraday rotation angle by the frequency spectrum analysis

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ABSTRACT

A novel method for measuring Faraday rotation angle by the frequency spectrum analysis is presented in this paper. The theoretical analysis and the experimental results demonstrate that Faraday rotation angle can be determined by the amplitude ratio of the signal's fundamental frequency to double frequency. A solenoid working on different voltages is used to produce different magnetic fields. The Faraday rotation angles in these magnetic fields have been measured. The linear fitting degree of the voltage and Faraday rotation angle is better than 99%.

1. Introduction

In the past years, Faraday effect has been widely applied in the current sensor, optical isolator, magneto-optical modem and analysis of chemical composition, etc. According to Faraday effect, when linearly polarized light is passing through Faraday material, the plane of polarization is rotated by the magnetic field. The measurement of Faraday rotation angle is the core in many applications. Always, the value of Faraday rotation angle wants to be required in the measurement of the magnetic field intensity and the measurement of the current for high voltage, etc.

Some measurement methods for Faraday rotation angle were demonstrated, such as the general Malus law method [1], AC modulation approximate calculation method [2] and double beams differential method [3]. On the basis, some researchers focused on new optical mediums [4–8] and new optical paths [9–12] to increase the accuracy of Faraday rotation angle measurement and designed some novel current sensors [13,14].

By the general Malus law method, the device includes a machine to rotate the analyzer. There are a lot of problems in the precision and the tenure of using for the device. AC modulation approximate calculation method only can be used to measure small Faraday rotation angle because the influence of high odd harmonics on the measurement results cannot be eliminated totally [15]. Moreover, the detection of double beams differential method needs a lot of optical material and usually has complicated optical design. In this paper, we find that the signal of frequency spectrum analysis can be used to measure Faraday rotation angle based on AC modulation. By this method, only two different frequency components of the output signal are used to calculate, so other high odd harmonics have no influence on the results of the measurement. This method can be used to measure big Faraday rotation angle. And, the device of the method tends to simplicity. The feasibility of the Faraday rotation angle detection method has been demonstrated in the paper.

2. Principle

According to the Faraday effect, the plane of polarization of a linearly polarized light beam is rotated under the influence of a magnetic field. The Faraday rotation angle Φ is given by

$$\Phi = \int_0^l VB\ dl \tag{1}$$

where V is the Verdet constant of the material, l is the material's length along the direction of magnetic field and B is magnetic field intensity.

For an alternate magnetic field, Φ becomes

$$\Phi = \int_0^l VB_0\ \sin \omega t\, dl = \Phi_0 \sin \omega t \tag{2}$$

where Φ_0 is the amplitude of Faraday rotation angle.

In Malus law, the intensity of a linearly polarized light beam through an analyzer is given by

$$I_0 = I_i\ \sin^2(\alpha + \Phi_0 \sin \omega t) \tag{3}$$

where I_i is the intensity of input light beam, α is the angle between the polarization azimuth of the linearly polarized light beam and the axis of analyzer passing through the orthogonal position.

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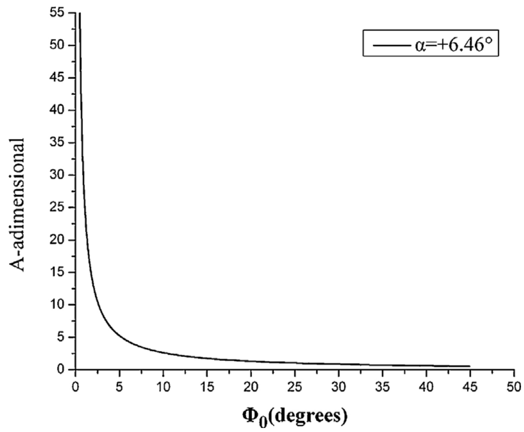

Fig. 1. The simulation relationship between the amplitude ratio (A) of fundamental frequency to double frequency and Faraday rotation angle, (Φ_0) when α is equal to 6.46°.

From Eq. (3), we can get

$$I_0 = I_i \frac{1-\cos(2\alpha + 2\Phi_0 \ \sin \omega t)}{2} = \frac{I_i}{2}[1-\cos(2\Phi_0 \sin \omega t)\cos 2\alpha + \sin(2\Phi_0 \sin \omega t)\sin 2\alpha] \quad (4)$$

According to Bessel equation, we can get

$$\cos(2\Phi_0 \sin \omega t) = J_0(2\Phi_0) + 2\sum_{n=1}^{\infty} J_{2n}(2\Phi_0)\cos 2n\omega t$$
$$\sin(2\Phi_0 \sin \omega t) = 2\sum_{n=1}^{\infty} J_{2n-1}(2\Phi_0)\sin(2n-1)\omega t \quad (5)$$

where n is the integer, and $J_0(2\Phi_0)$ is the DC component.

From Eqs. (4) and (5), I_0 can be calculated as

$$I_0 = \frac{I_i}{2}\left\{1-\left[J_0(2\Phi_0) + 2\sum_{n=1}^{\infty} J_{2n}(2\Phi_0)\cos 2n\omega t\right]\cos 2\alpha + \left[2\sum_{n=1}^{\infty} J_{2n-1}(2\Phi_0)\sin(2n-1)\omega t\right]\sin 2\alpha\right\} \quad (6)$$

Eq. (6) shows that the output signal I_0 contains not only fundamental frequency signal, but also double frequency signal and many high frequency signals. When the intensity of input light beam I_i is unchanged and α is a constant, the curve which is made sure by Eq. (3) only changes with the value of Φ_0. For a certain Φ_0, the output signal I_0 is calculated by Eq. (3) to make Fast Fourier transform (FFT). The amplitude ratio of fundamental frequency ($\omega/2\Pi$) to double frequency (ω/Π) in the frequency spectrum of I_0 is recorded as A. The intensity of input light beam influences the amplitude of fundamental frequency and double frequency with the same rate. So it does not influence the ratio A. This means that when α is a constant, the value of A is only changed with the value of Φ_0. We evaluate Φ_0 from 0° to 45° with step length being 0.01° and calculate the ratio A. The simulation relationship between A and Φ_0 when α is equal to 6.46° is shown in Fig. 1 which is a monotone decreasing curve. So, it is demonstrated that Faraday rotation angle can be found as soon as the values of A and α are conformed.

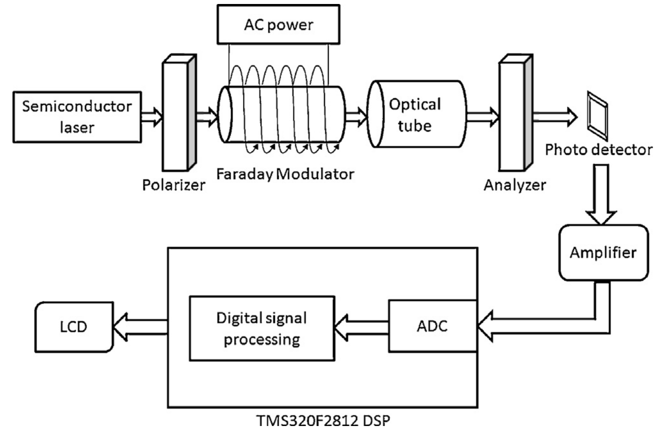

Fig. 2. The Faraday rotation angle measurement system.

3. Experiment and results

3.1. The analysis of the experiment system

Fig. 2 shows the experimental arrangement of the Faraday rotation angle measurement system. A 532 nm semiconductor laser is used as the light source and a polarizer is used to get a linearly polarized light beam. The Faraday modulator, which works under the 50 Hz AC power, is a solenoid which is wound by a current-carrying conductor. There is a bulk glass as sensing element in the Faraday modulator. The axis of analyzer is orthogonal to that of polarizer. The photo detector converts the light signal into the electrical signal. The electrical signal is amplified by an amplifier and sampled by an analog-digital converter (ADC). And, DSP (Digital Signal Processing) makes FFT to the signal which is converted by ADC. Finally, the results of the experiment display on a LCD. We do our experiment in a thermostatic chamber in which the temperature keeps at 20 °C.

The magnetic field intensity in the coil which is produced by a solenoid is given by

$$B = \mu_0 nI \quad (7)$$

where μ_0 is the magnetic conductivity permeability in a vacuum, n is the number of turns in the coil in the unit of length, I is the current in the conductor.

Keeping the frequency of the AC power unchanged, the current of the solenoid which is proportional to the voltage (U) of the power is given by

$$I = \frac{U}{R(\omega)} \quad (8)$$

where $R(\omega)$ is the impedance of the solenoid which is a constant as the frequency of the power is certain.

From Eqs. (2), (7) and (8), Φ_0 is proportional to the voltage of the power for a solenoid. Put a standard optical tube in the Faraday rotation angle measurement system. The standard optical tube can rotate the linearly polarized light +6.46° under the 532 nm light. The specific rotation of the tube dependency of the light wavelength can be written as [16]

$$[\alpha] = -0.1963657 + \frac{7.262667}{\lambda^2} + \frac{0.1171867}{\lambda^4} + \frac{0.0019554}{\lambda^6} \quad (9)$$

where $[\alpha]$ is the specific rotation of the tube which is proportional to the rotation angle made by the tube, and λ is the wavelength of the light.

Fig. 3 shows the change of the plane of polarization of a linearly polarized light beam.