Optik 123 (2012) 624–628

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Diffraction of plane waves by the interface between black and soft/hard semi-planes

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ARTICLE INFO

Article history:
Received 2 December 2010
Accepted 20 May 2011

Keywords:
Black screen
Diffraction theory
Fresnel integrals

ABSTRACT

A theory of scattering, based on the non-perturbation of the incident field, is developed for the black bodies. The method is applied to the diffraction problem of plane waves by an interface between the black and soft/hard half-planes. The solutions are obtained in terms of infinite series and then transformed into Fresnel integrals. The scattered fields are investigated numerically.

1. Introduction

A black body can be defined as an object which does not reflect or transmit the incoming waves on it. The scattering problem of waves by such a body has important applications in optics and electromagnetics. They represent the scattering characteristics of an object that is invisible to the radars.

There are generally three approaches for the modeling of the boundary conditions on a black screen. The first one is the Kirchhoff method. He assumed that the value of the field on the illuminated side of the screen is equal to that of the incident wave [1]. The field value on the shadow surface is zero. Although this approach was providing a convenience in the integral solution of the scattering problems, the overall formulation of Kirchhoff was inconsistent in terms of the boundary conditions [2]. Kottler defined jumped boundary conditions in order to show that Kirchhoff's formulation was the exact solution of a special class of problems [3,4]. Later, Asvestas proved that Kottler's approach was incorrect for the vector fields [5].

The second type of definition of the boundary conditions for a black screen is the approach of the impedance or mixed boundary conditions. This formulation is put forward by equating the ratio of the impedances of the surface and free space to the sine of the angle of incidence [6,7]. Although widely used in the literature, the conditions are strictly dependent on the angle of incidence [8].

The third approach is the Macdonald's method, which defines the black screen as the sum of the soft and hard screens [9,10]. This approach leads to the correct behavior of a black screen at high frequencies [11]. Ufimtsev showed the relation between Macdonald and Riemann black screens [12] and put forward the concept of the shadow radiation based on this method [13].

It is also possible to define special surface currents, which have no contribution on the geometrical optics (GO) fields, in the context of the modified theory of physical optics (MTPO) [14,15]. The stationary phase contributions of these integrals are equal to zero. If the black surface is discontinuous, there will be edge point contributions of the MTPO integrals which lead to the diffracted waves.

This study aims to investigate the scattering characteristics of screens, which are formed of the combinations of the black and soft/hard half-planes. We will derive boundary conditions for a black screen by using a similar approach to that of Macdonald's. The solutions will be obtained in terms of infinite series. The series will be transformed into Fresnel integrals and the scattered fields will be investigated numerically.

A time factor of $\exp(jwt)$ will be taken into account and suppressed throughout the paper. w is the angular frequency.

2. Theory

A black screen does not reflect or transmit the incident wave. Thus the incident will not be perturbed by the obstacle. A whole black plane, which is placed at the surface of $S = \{(x, y, z); x \in (-\infty, \infty), y = 0, z \in (-\infty, \infty)\}$, is taken into account. It is obvious that there is not any discontinuity in the geometry. The

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0030-4026/$ – see front matter
doi:10.1016/j.ijleo.2011.06.006

incident wave is approaching the screen from the half-plane of $y>0$. The total field can be written as

$$u_t = u_i \tag{1}$$

for $y>0$. u_i is the incident field. If the screen is a soft surface, the total field will be equal to zero. In this case, Eq. (1) reads

$$u_{ts} = u_i + u_{rs} \tag{2}$$

where u_{rs} is the reflected GO field on a soft surface. The total field can be expressed as

$$u_{th} = u_i + u_{rh} \tag{3}$$

for a hard surface on which the normal derivative of the total field is equal to zero. u_{rh} is the reflected field for the hard surface. Note that the relation of

$$u_{rs} + u_{rh} = 0 \tag{4}$$

exists between the reflected waves. Thus Eq. (1) can be rewritten as

$$u_t = u_i + \frac{u_{rs}}{2} + \frac{u_{rh}}{2} \tag{5}$$

which leads to the expression of

$$u_t = \frac{1}{2}(u_i + u_{rs}) + \frac{1}{2}(u_i + u_{rh}). \tag{6}$$

The boundary conditions of

$$u_t|_S = \frac{1}{2}(u_i + u_{rh})\Big|_S \tag{7}$$

and

$$\left.\frac{\partial u_t}{\partial n}\right|_S = \frac{1}{2}\frac{\partial}{\partial n}(u_i + u_{rs})\Big|_S \tag{8}$$

can be obtained on the surface of the black plane. n is the unit normal vector of the surface. Eq. (6) can be rewritten as

$$u_t = \frac{1}{2}(u_{ts} + u_{th}) \tag{9}$$

for u_{ts} and u_{th} are defined by Eqs. (2) and (3), respectively. Eq. (9) is equivalent to the approach of Macdonald [9,10]. Eqs. (7) and (8) are the alternative representation of the conditions of Macdonald. The solution of a scattering problem by a black body can be performed according to Eq. (9) as follows: (1) the problem is solved by soft and hard surfaces for the same geometry and (2) the two solutions are added and divided by two.

3. Scattering of plane waves by the interface between black and soft/hard half-planes

The discontinuity of the problem occurs at the interface of the two half-planes with different boundary conditions. The GO waves will suddenly change and there will be diffracted waves which compensates the discontinuities of the GO fields. The general solution of the Helmholtz equation can be given by the expression of

$$u_t = J_\nu(k\rho)(A_\nu \sin\nu\phi + B_\nu \cos\nu\phi) \tag{10}$$

in the cylindrical coordinates [16]. ν is the constant of separation. A_ν and B_ν are the constant coefficients that will be determined by the boundary conditions on the surface. The incident plane wave can be represented as

$$u_i = J_0(k\rho) + 2\sum_{n=1}^{\infty} J_n(k\rho)e^{jn(\pi/2)}\cos n(\phi-\phi_0) \tag{11}$$

according to the generating function of the Bessel functions. We will examine four cases in this context.

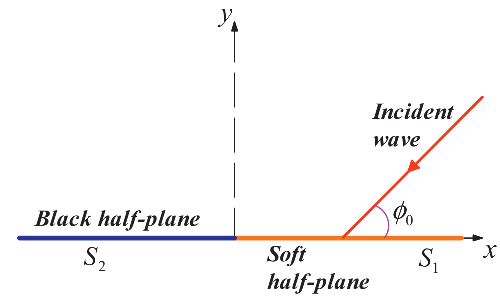

Fig. 1. Geometry of the problem.

3.1. Soft-black half-planes

The surface, at $\phi=0$ (S_1), is a soft half-plane. The semi-plane, at $\phi=\pi$ (S_2), is a black surface. The incident plane wave, which has a unit amplitude, hits the surface with an angle of incidence of ϕ_0. The geometry of the problem is given in Fig. 1. Since S_1 is a soft surface, B_ν is equal to zero according to the boundary condition of

$$u_t|_{\phi=0} = J_\nu(k\rho)(A_\nu 0 + B_\nu 1) = 0. \tag{12}$$

A_ν is found to be

$$A_\nu = 4e^{j\nu(\pi/2)}\sin\nu\phi_0 \tag{13}$$

when Eq. (10) is compared with Eq. (11). Now we will evaluate u_{ts} and u_{th}. The boundary conditions of

$$u_{ts}|_{\phi=\pi} = A_\nu J_\nu(k\rho)\sin\nu\pi = 0 \tag{14}$$

and

$$\left.\frac{\partial u_{th}}{\partial\phi}\right|_{\phi=\pi} = A_\nu J_\nu(k\rho)\nu\cos\nu\pi = 0 \tag{15}$$

can be written. Thus the fields of u_{ts} and u_{th} are found to be

$$u_{ts} = 4\sum_{n=1}^{\infty} J_n(k\rho)e^{jn(\pi/2)}\sin n\phi \sin n\phi_0 \tag{16}$$

and

$$u_{th} = 4\sum_{n=0}^{\infty} J_{\alpha_n}(k\rho)e^{j\alpha_n(\pi/2)}\sin\alpha_n\phi \sin\alpha_n\phi_0 \tag{17}$$

respectively. α_n is equal to $(2n+1)/2$. The total scattered field is the half of the sum of u_{ts} and u_{th} according to Eq. (9). The GO solution of the problem can be given by the equation of

$$u_t^{GO} = e^{jk\rho\cos(\phi-\phi_0)} - e^{jk\rho\cos(\phi+\phi_0)}U(\pi-\phi_0-\phi). \tag{18}$$

$U(x)$ is the unit step function, which is equal to one for $x>0$ and zero otherwise.

Fig. 2 shows the variation of the total fields versus the observation angle. The observation distance is 6λ for λ is the wave-length. The angle of incidence is equal to 60°. The reflected GO field goes to zero at $\phi=\pi-\phi_0$ and only the incident field exists for $\phi>\pi-\phi_0$. The series solution is continuous everywhere. Thus it contains the diffracted wave which compensates the reflected field. The sum of the diffracted and incident waves gives the interference pattern, observed for $\phi>\pi-\phi_0$.