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Wavefront fitting and comparison of camera aberrations using Zernike circle polynomials

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ABSTRACT

Zernike polynomials are in widespread use for wavefront analysis because of their representation of classical aberrations. This paper presents a method to investigate wavefront aberrations which does not require complicated and expensive wavefront sensors. Images of checkerboard pattern are captured using mobile phone cameras, CCTV camera, Firewire camera, Handy cam and Stereo camera and wavefront is constructed using displacement in the grid corners of checkerboard patterns. This wavefront is fitted into Zernike circle polynomials and Zernike coefficients are computed using Gaussian elimination method. The computed Zernike polynomials and Zernike coefficients are further utilized to compare the aberrations of different cameras. The experiment and simulations confirm that displacement in the grid corners of the checkerboard pattern can be treated as a wavefront and fitted to Zernike polynomials. Stereo camera is also investigated for both raw and rectified set of images.

1. Introduction

Optical imaging systems have aberrations due to the imperfections in the lenses and other factors [1]. Spherical aberrations distort the information present and reduce the maximum resolution obtainable by the imaging system. To improve the image quality digitally camera aberrations should be characterized quantitatively.

Zernike polynomials for circular aperture are useful for quantitatively characterizing aberrated wavefront of an optical system. Noll introduced the orthonormal form of Zernike polynomials [2] and used them to describe the aberrations of an optical wave propagating through Kolmogorov atmospheric turbulence. Fried [3] used a form of these polynomials to describe the statistical strength of aberrations produced by atmospheric turbulence. Bezdidko [4] discussed the advantages of Zernike polynomials in solving many optical problems. Mahajan [5–7] studied the optical aberrations of systems with circular pupils using orthonormal form of Zernike circle polynomials and Zernike annular polynomials for imaging systems with annular pupils.

In this paper we use Zernike circle polynomials for calculating and comparing the aberrations of different types of cameras such as camera on Mobile phones, off the shelf available CCTV camera, Firewire camera, Stereo camera and Handy cam. Images of checkerboard pattern of size (20×20) squares and (30×30) squares are captured using these cameras. Checkerboard patterns are circumscribed to a unit circle (Fig. 1 (a)) and used to obtain Zernike polynomials coefficients. The original checkerboard pattern free of distortions is termed as the 'standard image' and the distorted checkerboard patterns captured using these cameras are termed as 'observed image'. The displacement in the grid corners of the checkerboard pattern of the observed image with respect to standard image represents the wavefront. The aberrations in the wavefront are estimated by fitting Zernike circle polynomials and their coefficients are determined using least squares Gaussian elimination method [8]. Comparison of camera aberrations is done in terms of the Zernike coefficients. Mitsuhashi and Tadano [9] measured the wavefront distortion caused by thermal deformation of SR extraction mirror by means of Hartmann Screen Test.

The remainder of this paper is organized as follows. Section 2 presents wavefront extraction and fitting. Section 3 describes the method adopted for determination of Zernike coefficients. Section 4 describes the experiment and computer simulations of the algorithm and provides the assertion for considering displacement in the grid corners as wavefront. It also illustrates the limitations of checkerboard resolution and the dependence of camera aberrations on lens specifications. Section 5 gives experimental results on images of checkerboard pattern and Section 6 concludes the paper.

2. Wavefront extraction and fitting

Checkerboard patterns of size (20×20) squares and (30×30) squares are captured using different cameras. Grid corners of the

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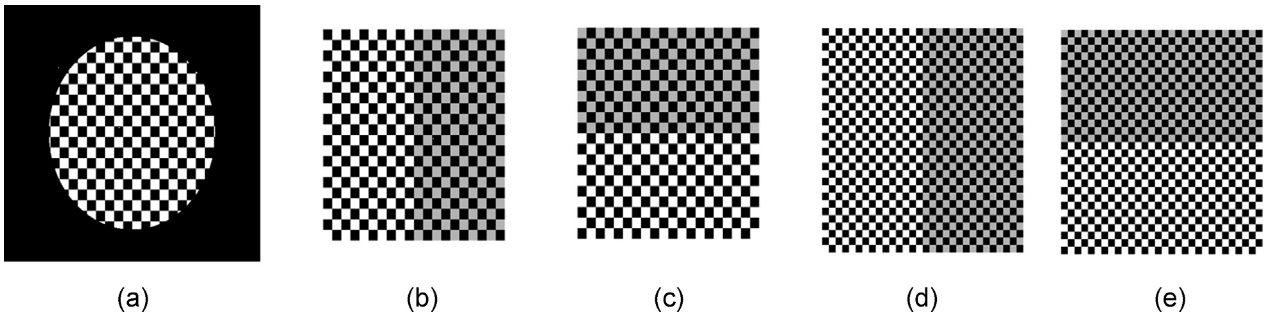

Fig. 1. Standard images: (a) checkerboard pattern of size (20 × 20) squares circumscribed to a unit circle; (b) and (c) straight and 90° rotated checkerboard pattern of size (20 × 20) squares; (d) and (e) straight and 90° rotated checkerboard pattern of size (30 × 30) squares.

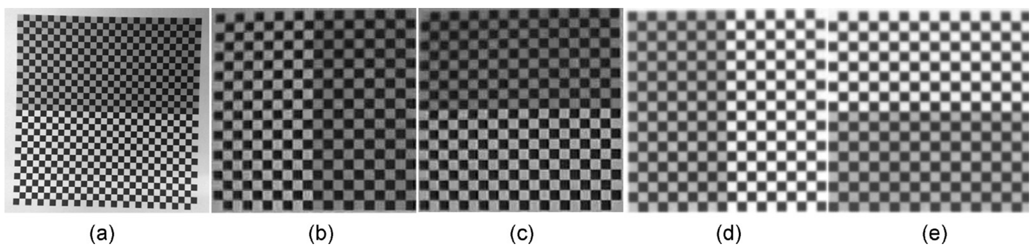

Fig. 2. Observed images: (a) 90° rotated checkerboard pattern of size (30 × 30) squares captured by one of the mobile phone cameras; (b) and (c) straight and 90° rotated checkerboard pattern of size (20 × 20) squares captured by CCTV camera; (d) and (e) straight and 90° rotated checkerboard pattern of size (20 × 20) squares captured by Firewire camera.

standard and observed checkerboard image are determined. The displacement in the grid corners of the standard and observed checkerboard pattern is termed as the wavefront.

Zernike polynomials are a set of orthogonal polynomials defined on a unit circle.

$$\left.\begin{array}{ll} \left.\begin{array}{l} Z_{\mathrm{even}j}(r,\theta)=[2(n+1)]^{1/2}R_n^m(r)\cos m\theta \\ Z_{\mathrm{odd}\,j}(r,\theta)=[2(n+1)]^{1/2}R_n^m(r)\sin m\theta \end{array}\right\} & m\neq 0 \\ Z_j(r,\theta)=[(n+1)]^{1/2}R_n^m(r) & m=0 \end{array}\right\} \quad (1)$$

where the radial polynomial $R_n^m(r)$ is given as:

$$R_n^m(r)=\sum_{s=0}^{(n-m)/2}\frac{(-1)^s(n-s)!}{s![(n+m)/2-s]![(n-m)/2-s]!}r^{n-2s} \quad (2)$$

The indices n and m are the radial degree and the azimuthal frequency respectively and should satisfy $m \le n$ and $n-m=$ even, j is a function of n and m called mode ordering number. Orthonormal Zernike circle polynomials $Z_j(r, \theta)$ and their popular names are widely known in literature [6]. These Zernike polynomials are suitable for estimating aberrations of circular aperture imaging systems. Dai and Mahajan [10] emphasized the use of orthonormal polynomial for non-circular pupils, since Zernike polynomials do not presents balanced aberrations for a noncircular pupil. By applying Gram-Schmidt orthogonalization technique another form of Zernike polynomials can be obtained known as Zernike annular polynomials [11].

Wavefront fitting procedure expands a wavefront phase into a series of wavefronts which follow the properties of polar (r, θ) orthogonal system over the area of interest (here unit circle) and it has to be complete in the sense that any function can be approximated by linear combinations of the wavefront series with any degree of accuracy [12]. Since Zernike polynomials follow these properties and form a complete set, any

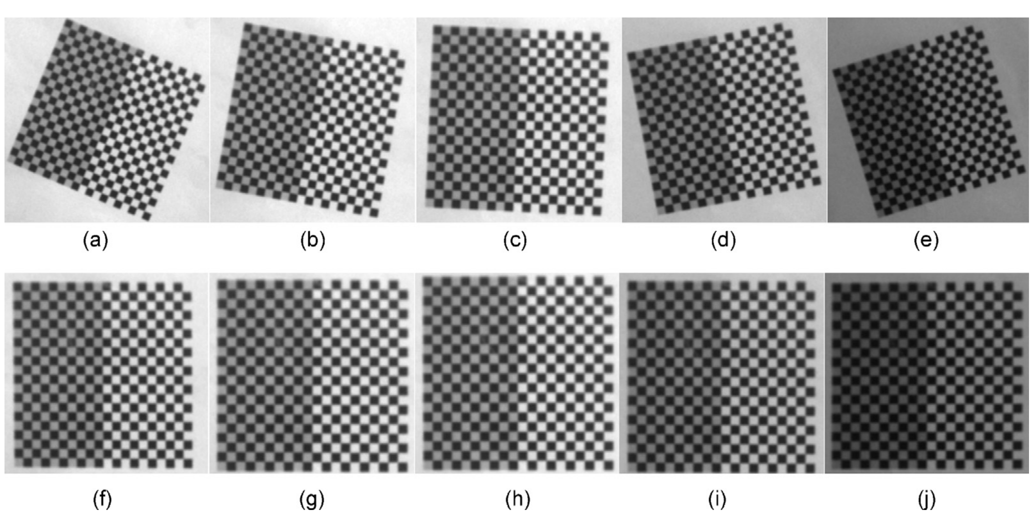

Fig. 3. (a)–(e) Images captured from different viewpoints by Firewire camera; (f)–(j) affine transformed images of images shown from (a)–(e).