Optik 124 (2013) 590–606

Contents lists available at SciVerse ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Extracting parameters from slanted non-uniform gratings recorded in photopolymer. Part I: Theoretical derivations

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ARTICLE INFO

Article history:
Received 26 August 2011
Accepted 21 December 2011

Keywords:
Non-uniform grating
Slanted gratings
Photopolymer
Coupled-wave theory
Volume holography

ABSTRACT

Despite the practical significance of slanted volume holographic gratings, most research presented in the photopolymer literature involves the use of unslanted reflection or transmission geometry gratings. A physically accurate electromagnetic model of the slanted holographic non-uniform gratings recorded in photopolymers is necessary in order to extract key volume grating parameters. In this paper we present a model, based on a set of two coupled differential equations, which include the effects of: (i) an exponential decay of refractive index modulation in the direction of the beam propagation due for example to the effects of dye absorption with depth; (ii) Gaussian profile of refractive index modulation due to recording by finite Gaussian beams, and (iii) a quadratic variation in the spatial period of the grating (chirp) arising due to non-uniform average index and thickness changes, i.e., shrinkage and swelling. Analytic results and numerical simulation are presented. In Part II the model developed in Part I is applied to fit experimental data, i.e., angular scans, of slanted gratings recorded in a polyvinylalcohol/acrylamide based material for different slant angles in order to extract key volume grating parameters.

1. Introduction

Photopolymers are non-latent self-processing recording materials, i.e., they directly respond during exposure at a sensitised wavelength without the need for post-processing [1–4]. Even if a constant illuminating pattern (amplitude and spatial period) is used during exposure, non-uniform gratings are in general recorded due to the material processes taking place during grating formation [1,2,4–10]. We examine the case of a two plane wave interference pattern exposure of a photopolymer layer. The grating typically begins to form a few seconds after the beginning of the exposure (delayed in some cases due to inhibition [2,7]) and thereafter continues to grow. The grating may also evolve post-exposure due to dark effects [2,11–13]. In the material layer photon absorption and the associated photo-chemical reactions lead to primary radical production, which initiates polymer chain formation in the bright (exposed) regions [2,7,11–16]. It is known that in non-latent materials the presence of the growing grating can affect the exposing beams propagating through the volume, altering the interference pattern locally, over time and leading to changes in the grating shape recorded in the photopolymer volume [1,2,4]. Grating shape non-uniformities can also be caused by material non-linearity [1,2,7,17], photosensitiser absorption [18–20], the profiles of the exposing beams [18], as well as thickness variations of the layer with time. All of these effects give rise to non-uniform monomer and polymer distributions in the material, resulting in average refractive index variations and changes in the grating shape and period within the layer [4,8–10,18,21].

In this two part paper we examine sinusoidal exposure of a polyvinylalcohol/acrylamide (PVA/AA) photopolymer layer [7] to produce transmission geometry volume gratings. At the start of the exposure the photosensitiser (dye) absorbs most light at the front (or input) side of the photopolymer layer (where most photons are present). Excited dye dissociates and produces free radical, which, if inhibition effects do not take place, initiates the polymerisation process. As the monomer (acrylamide) is used up it is converted into the denser polymer form (polyacrylamide) which has a higher value of refractive index. As this process continues a monomer concentration gradient arises. This leads to monomer diffusion into the exposed regions (i.e., from the dark fringes and from the back of the material layer). Such monomer diffusion can contribute to producing a higher value of the grating refractive index modulation [22]. The larger number of photons initially

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0030-4026/$ – see front matter © 2012 Elsevier GmbH. All rights reserved.
doi:10.1016/j.ijleo.2012.01.004

absorbed at the front of the layer, and the greater the amount of monomer that diffuses into the bright regions, the larger the variations in the refractive index changes generated between the front and back of the layer. The rate, at which polymerisation takes place, depends on the availability of the reactants (dye, co-initiator, monomer) and on the exposing intensity. Furthermore as the exposure takes place, the location at which maximum polymerisation takes place, will change within the material layer as for example the position at which the maximum dye concentration remaining available to absorb changes from the front input to the back of the layer [23].

Other effects can also alter the recorded grating uniformity with depth are the volume changes, i.e., shrinkage/swelling, which can take place as a result of polymerisation and diffusion in the layer. The creation of (denser) polymer will in general lead to localised shrinkage. Uncovered (unsealed) photopolymer layers can interact with the surrounding environment, e.g., absorbing water, which eventually leads to swelling [24–26]. These changes can introduce surface variations and optical path variations: (i) grating fringe spacing, i.e., varying the grating period with depth, and (ii) average index changes with depth. All these effects can lead to the Bragg replay conditions, i.e., the electromagnetic coupling (volume diffraction strength) within the grating, varying with depth. Thus the incident light may be diffracted with different efficiencies at different depths, and the Bragg condition is a function of the position within the layer.

In this paper a set of first-order coupled differential equations describing diffraction by non-uniform gratings are presented and solved. We note however that dynamical non-latent effects are not taken into account [4]. In order to verify the model, in Part II results for slanted volume gratings recorded in PVA/AA material layers are reported with off-Bragg angular scans measured. Best fits to the resulting experimental data, using the non-uniform diffraction model, are performed and the grating parameters which give the best fits to these data curves are identified. The quality of these fits is quantified using the root mean square error (RMSE).

The paper is organised as follows in Section 2 starting with Maxwell equations, the differential equations governing diffraction by a thick lossless grating are derived. In Section 3 the effects of several types of grating non-uniformities are introduced in the model, these include: (i) index modulation with depth, and (ii) variations of the on-Bragg replay condition with depth. In Section 4 approximate analytical solutions are presented and compared in Section 5 with more physically exact numerical solutions and the diffraction effects of the non-uniformities are discussed. A brief conclusion is presented in Section 6.

2. Derivation of coupled-wave equations

Maxell's differential equations governing wave propagation in a lossy dielectric materials with no free charges are [1]:

$$\nabla \cdot (\varepsilon \vec{E}) = 0, \tag{1a}$$

$$\nabla \cdot (\mu \vec{H}) = 0, \tag{1b}$$

$$\nabla \times \vec{E} = -\frac{\partial(\mu \vec{H})}{\partial t}, \tag{1c}$$

$$\nabla \times \vec{H} = \sigma \vec{E} + \frac{\partial(\varepsilon \vec{E})}{\partial t}, \tag{1d}$$

where $\vec{H}$ and $\vec{E}$ are the vector of magnetic intensity and electric intensity, respectively, σ is the material conductivity, which characterises the optical absorption or loss of the material, $\mu = \mu_0$ is the magnetic permeability for free space, (no ferromagnetic material: $\mu_r = 1$) and lastly the electric permittivity of material, $\varepsilon = \varepsilon_r \varepsilon_0$, where ε_0 and ε_r are the permittivity of free space and the relative permittivity of the particular medium respectively. Assuming a monochromatic field propagating through the medium

$$\vec{E}(x, y, z, t) = \vec{E}(x, y, z) \exp[j\omega_0 t], \tag{2a}$$

and

$$\vec{H}(x, y, z, t) = \vec{H}(x, y, z) \exp[j\omega_0 t], \tag{2b}$$

where $\omega = 2\pi f_0$ is the angular frequency and f_0 is the frequency of the light. Substituting from Eq. (2a) into Eqs. (1a) and (1b) gives that

$$\nabla \times \vec{H}(x, y, z) = \sigma \vec{E}(x, y, z) + j\omega_0 \varepsilon \vec{E}(x, y, z), \tag{3a}$$

and

$$\nabla \times \vec{E}(x, y, z) = -j\omega_0 \mu \vec{H}(x, y, z) \tag{3b}$$

Combining Eqs. (3a) and (3b), i.e., taking the curl of Eq. (3b) and substituting from Eq. (3a), we eliminate magnetic intensity and arrive at the time independent vector wave equation

$$\nabla \times [\nabla \times \vec{E}(x, y, z)] + (j\omega_0 \mu \sigma - \omega_0^2 \mu \varepsilon_0 \varepsilon_r) \vec{E}(x, y, z) = 0 \tag{4}$$

Noting that $\nabla \times (\nabla \times \vec{E}) = \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E}$, and assuming that the E-vector is perpendicular to the plane of interest, i.e., there is no material variation in the y direction as illustrated in Fig. 1, Eq. (4) reduces to a scalar wave equation

$$\nabla^2 E_y(x, z) + \beta^2 \left(1 - j\frac{\varepsilon'_{av}}{\varepsilon_{av}}\right) E_y(x, z) = 0, \tag{5}$$

where $\beta = 2\pi n_{av}/\lambda$ is the angular frequency of the propagating wave in the material, $n_{av} = \sqrt{\varepsilon_{av}}$ and λ is the wavelength of the light in free space. ε_{av} and ε'_{av} are the average real and imaginary parts of the material relative ε_r permittivity respectively, i.e., $\varepsilon_r = \varepsilon_{av} - j\varepsilon'_{av}$. We note that

$$\varepsilon'_{av} = \frac{\sigma}{\varepsilon_0 \omega_0}, \tag{6a}$$