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Stability investigation on quantum correlations of coupled qubits in non-Markovian process

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ABSTRACT

Utilizing the concurrence and the quantum discord as the measure method, in this paper we compare and investigate the dynamic evolution features of quantum correlations of coupled qubits in non-Markovian process. We focus attention on decoherence effect influences the stability of quantum correlations. The investigation results show that because of the decoherence influence between the system and environment, the concurrence always evolves with time in oscillation form in the way of deaths and survivals, however, the quantum discord time evolution does not appear the deaths and survivals. The quantum discord survives obviously longer than concurrence, which indicates that quantum discord has a stronger ability to resist decoherence than entanglement. Through regulating and controlling the purity and entanglement of the initial quantum state, we can effectively suppress the decay of the quantum correlations, which is advantaged to complete the quantum information processing.

1. Introduction

Superconducting qubit is the solid quantum circuit based on the integrated circuit craft. With the characters of low dissipation and scalability, it is the most promising project to realize the quantum computing [1,2]. From the 1980s, people sequentially observed many kinds of macroscopic quantum phenomena in superconducting qubits, which shows that the superconducting qubit is equivalent to the artificial atoms or artificial spin, and we can easily control, couple and measure the artificial atoms or artificial spin due to their macroscopic scale, this is not only a favorable factor for quantum information processing but also important for us to research the problems in the field of the atom physics and the quantum optics to realize the strong coupling limit which is impossible for the natural atom system. Considering the significant meaning in aspects of basic theory and potential applications, people pay wide attention to the superconducting qubits. Recently people have made large progresses in decoherence research and have achieved the basic requirements of quantum information processing. Furthermore, the large design and process freedom of the superconducting qubit provides more choices for the scale, and it is possible to make real breakthrough in quantum computing aspect.

In order to complete the quantum information processing, keeping the quantum correlations between the qubits is necessary. For a period of time people just thought that the entanglement between qubits was an important material of quantum information and decoherence effect could cause disentanglement or even the entanglement death in coupled qubits. Therefore, in the recent ten years, people have made numbers of researches theoretically and experimentally on the problems of entanglement measure and entanglement keep and have achieved large progresses [3–8]. The researchers found that the quantum correlation was a more important quantum information resource than quantum entanglement. Entanglement was just a special existing form of quantum correlations. It was the most attractive progress had been made. And people also found that the separable states without entanglement might contain the non-classical correlations called the quantum correlations [9]. The exist of the quantum correlations without entanglement will not only provide more technology projects for selecting to realize the quantum information processing, but also greatly expand the targets and ranges of the quantum information processing. Recently, Datta proposed a model that could realize the quantum speedup algorithm without depending on the quantum entanglement. In their model, any entanglement resource was used and the state was throughout separable state. We can also see that if it can realize the quantum algorithm without the help of entanglement, then we would not need to excessively worry about the disentangled effect. In this way it can greatly decrease the experiment difficulties and the equipment requirements. Presently, the

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quantum discord proposed by Olliver and Zurek is used for measuring the quantum correlations [10]. Under this measure way and in the Markovian process, people have made comparisons and investigations on the entanglement and the quantum discord of some weak coupling qubits system, and obtained some interesting results [11,12].

For solid quantum system such as the superconducting qubits, the interaction between the system and the environment is generally strong. The dynamic evolution of the quantum system is a typical non-Markovian process. On the other hand, even for some open quantum systems that can be described by Markovian approximation, the Markovian approximation actually only describes the long time evolution behaviors of the system. For the initial evolution stage, the non-Markovian model should be used to describe the open quantum systems. The main character of the quantum system in the non-Markovian process is that the effects the system previously affected the environment can feedback to the system and affect the system current state. If we do not consider this feedback effect, the system dynamic evolution behaviors would not be exactly described. In this paper, through establishing proper quantum master equation and applying quantum entanglement and quantum discord, we will make comparison and investigation on the dynamic behaviors of the quantum correlations of coupled qubits in non-Markovian process, and discuss the projects that controlling the entanglement and quantum discord.

2. Model

The superconducting circuit model in Fig. 1 we investigate has been researched in many experiments. In Fig. 1, the two single Cooper pair boxes (CPBs) are connected by a common current-biased Josephson junction (CBJJ). Each qubit consists of a gate electrode of capacitance C_g and a single Cooper pair box. And two Josephson junctions have exactly the same capacitance C_J and Josephson energy E_J, which form a superconducting quantum interference device (SQUID) ring threaded by a flux Φ. The superconducting phase difference across the jth qubit is represented by Φ_j (j = 1,2). The CBJJ has capacitance C_b, phase drop θ_b, Josephson energy E_b, and a bias current I_b. V_g represents the gate voltage.

With the conditions of degeneracy and symmetry, based on the rotating-wave approximation, the Hamiltonian of the system can be simplified as [13–15]

$$H_S = \frac{1}{2}\omega_0\sigma_z^1 + \frac{1}{2}\omega_0\sigma_z^2 + J(\sigma_+^1\sigma_-^2 + \sigma_+^2\sigma_-^1), \tag{1}$$

where $E_{J1} = E_{J2} = \hbar\omega_0$, the coefficient J is the interaction strength between the two qubits.

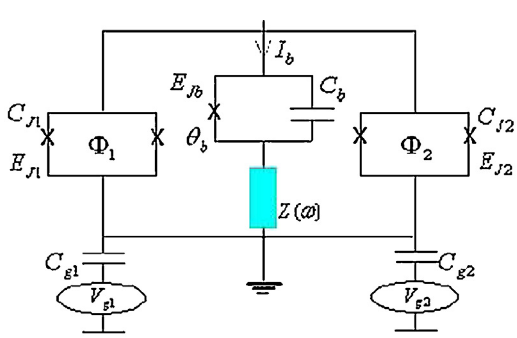

Fig. 1. Two Josephson charge qubits are controllably coupled to a common current-biased Josephson junction, which operates as a Josephson phase qubit and acts as a coupler. Two Josephson qubits and current-biased Josephson junction with the electromagnetic environment represented by the impedance $Z(\omega)$.

According to the quantum dissipation theory, in the non-Markovian process, the reduced density ρ of the coupling quantum system satisfies the following motion equation [16]

$$\frac{d}{dt}\rho(t) = -i[H_S,\rho] + \sum_{k=1}^{2}\left(\Gamma_1 D\left[\sigma_k^-\right]\rho + \Gamma_2 D\left[\sigma_k^+\right]\rho\right) \tag{2}$$

$\Gamma_1 = \Delta(t) + \gamma(t), \quad \Gamma_2 = \Delta(t) - \gamma(t)$, where

$$\Delta(t) = \int_0^t d\tau k(\tau)\cos(\omega_0\tau), \tag{3}$$

$$\gamma(t) = \int_0^t d\tau\mu(\tau)\sin(\omega_0\tau), \tag{4}$$

$$k(\tau) = 2\int_0^\infty d\omega J(\omega)\coth\left[\frac{\hbar\omega}{2k_BT}\right]\cos(\omega\tau), \tag{5}$$

$$\mu(\tau) = 2\int_0^\infty d\omega J(\omega)\sin(\omega\tau). \tag{6}$$

For simplicity, in this paper we discuss the question only in the Ohmic environment. Therefore, we choose the Ohmic spectral density as

$$J(\omega) = \frac{2\gamma_0}{\pi}\omega\frac{\omega_c^2}{\omega_c^2 + \omega^2}. \tag{7}$$

where γ_0 is the frequency-independent damping constant, ω is the frequency of the bath, and ω_c is the high-frequency cutoff. The parameters $\gamma(t)$ and $\Delta(t)$ contain the non-Markovian features of the quantum system. With the help of the method of residues, the $\Delta(t)$ and $\gamma(t)$ are calculated by using the formula

$$\coth\left(\frac{\omega}{2k_BT}\right) = 2k_BT\sum_{n=-\infty}^{+\infty}\frac{\omega}{\omega^2 + \omega_n^2}, \tag{8}$$

where the $\omega_n = 2\pi nk_BT$ are known as the Matsubara frequencies. In the follows, we consider the case for a zero temperature environment. Then, under the given spectral density, we can get [17]

$$\Delta(t) = 0, \tag{9a}$$

$$\gamma(t) = \frac{\omega_0 r^2}{1 + r^2}(1 - e^{-r\omega_0 t}\cos(\omega_0 t) - re^{-r\omega_0 t}\sin(\omega_0 t)), \tag{9b}$$

with $r = \omega_c/\omega_0$.

For an arbitrary two-qubits state, especially in the non-Markovian process, it is usually difficult to obtain the numerical solutions of the quantum master equations and is even more difficult to gain the analytic solutions. In this paper, we will only discuss and analyze a kind of special X-state. When the system is initially in X-state, it can always keep in the X-state unchanged in the dynamic evolution process. The X-state is given as follows:

$$\rho_{AB}(t) = \begin{pmatrix} \rho_{11}(t) & 0 & 0 & \rho_{14}(t) \\ 0 & \rho_{22}(t) & \rho_{23}(t) & 0 \\ 0 & \rho_{32}(t) & \rho_{33}(t) & 0 \\ \rho_{41}(t) & 0 & 0 & \rho_{44}(t) \end{pmatrix}. \tag{10}$$

According to the quantum master equation (2), we can obtain the following two groups of equations, one of which describe the time evolution of the populations, namely

$$\dot\rho_{11}(t) = -2\gamma(t)\rho_{11}(t) - \gamma(t)(\rho_{22}(t) + \rho_{33}(t)), \tag{11a}$$

$$\dot\rho_{22}(t) = \gamma(t)\rho_{11}(t) - \gamma(t)\rho_{44}(t) - iJ(\rho_{32}(t) - \rho_{23}(t)), \tag{11b}$$

$$\dot\rho_{33}(t) = \gamma(t)\rho_{11}(t) - \gamma(t)\rho_{44}(t) + iJ(\rho_{32}(t) - \rho_{23}(t)), \tag{11c}$$