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Entanglement and sudden death of two two-level atoms interacting with a cavity field in presence of degenerate parametric amplifier

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ABSTRACT

In the presence of degenerate two-photon transitions the problem of the interaction between two two-level atoms and a single-mode is considered. Near resonance case, a closed form of the analytic solution for the wave function is obtained. The entanglement between an atom and field in the interacting system is studied by using the change in atomic and field entropies. The relationship between entropy changes and concurrence entanglement is discussed. Our results show that the behavior of the entropy change in agreement with the behavior of the concurrence to measure the entanglement between two subsystem structures.

1. Introduction

In the framework of quantum optical systems such as cavity quantum electrodynamics the concept of quantum entanglement has been introduced. Many different experiments were performed in recent years and several types of entangled states have been produced [1]. For example, a recent experimental breakthrough has been obtained by entangling two distant atomic qubits by their interaction with the same photon [2]. Furthermore it is also considered as an important resource for some nonclassical tasks such as quantum computation, quantum cryptography protocols and quantum teleportation, see for example Refs. [3–7]. In fact entanglement is responsible for the nonlocal correlations between spatially separated quantum systems as revealed by the violation of Bell's inequality [8]. Implementation of quantum information protocols is found to be useful in communication and computation. In fact quantification of entanglement is an important aspect that is realized in some studies to perform and quantify the entanglement during the interaction in an atom–photon system within cavities [9–14]. As is well known, the interaction between the environment and quantum systems can lead to decoherence. However, it can also induced entanglement. In this context the authors of Ref. [14] have shown that entanglement can always arise in the interaction of an arbitrarily large system in any mixed state with a single qubit in a pure state. This is emphasized during the study of the interaction between a two-level atom in a pure state and a field in a thermal state at an arbitrarily high temperature. Khalil et al. [15,16] have investigated the entanglement of two identical two-level atoms with a two-photon transition induced by a single-mode squeezed field. The same problem is also considered by Zhou et al. [12], however, for non-identical atoms. The same authors in a different communication considered the entanglement of two identical two-level atoms through a nonlinear two-photon system in interaction with a one-mode thermal field [17]. They have shown that the entanglement between two atoms induced by nonlinear interaction is larger than that induced by linear interaction. Therefore it would be of interest to investigate the entanglement between two identical atoms induced by thermal noise through different nonlinear processes. In reality we have to consider more than one atom within a cavity. For instance the interaction between three atoms and a single field leads to the phenomenon of superradiance (the collective spontaneous emission of a system of many atoms), in which the atoms are strongly coupled by their common interaction with a resonant electric field. In the case of three atoms the phenomenon begins to be exhibited. However, when the number of atoms is just two, the observation of radiation trapping has been reported. Most of the previous work restricted to the exact resonance case, this means that the discussion related to the problem is limited. Therefore in this communication we consider the same problem and extend our discussion to include the effect of

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detuning. This in fact gives us an opportunity to see the difference between the two cases and to examine the variation that would occur resulting from the detuning parameter. The Hamiltonian model which describes the problem of two two-level atoms in the presence of second harmonic generation and in interaction with the field assumes the form [18–22]

$$\frac{\hat{H}}{\hbar} = \omega\hat{a}^{\dagger}\hat{a} + \sum_{j=1}^{2}\Omega_j\hat{S}_z^{(j)} + i\ \ (\hat{a}^{\dagger 2} - \hat{a}^2)\sum_{i=1}^{2}\lambda_i(\hat{S}_+^{(i)} + \hat{S}_-^{(i)}) + \lambda_3(\hat{a}^{\dagger 2} + \hat{a}^2). \quad (1)$$

where ω is the field frequency, Ω_j is the atomic energy difference and λ_i, i = 1, 2 are the atom–field coupling parameters, while λ_3 is the coupling response for the second harmonic generation. The operators $\hat{a}^{\dagger}$ and $\hat{a}$ are the creation and annihilation operators, respectively, which satisfy the commutation relation $[\hat{a}, \hat{a}^{\dagger}] = 1$. The operators $\hat{S}_+^{(i)}$, $\hat{S}_-^{(i)}$ and $\hat{S}_z^{(i)}$ are the Pauli 1/2 spin matrices and obey the relations

$$[\hat{S}_+^{(i)}, \hat{S}_-^{(j)}] = \hat{S}_z^{(i)}\delta_{ij}, \quad [\hat{S}_z^{(i)}, \hat{S}_{\pm}^{(j)}] = \pm\hat{S}_{\pm}^{(i)}\delta_{ij}, \quad i, j = 1, 2. \quad (2)$$

It should be noted that in the above Hamiltonian we have taken all the coupling parameters to be constant. Thus the problem would be much easier and the probability to find the dynamic operators or the wavefunction becomes more tractable than for the nonconservative case. However, the existence of the nonlinearity in the interaction part would lead to some complications. Therefore to avoid such a complication we invoke a canonical transformation to remove the degenerate parametric term. This is done in the forthcoming section in which a closed-form solution of the wavefunction is also obtained. This is followed by a discussion of the change in the field and in the atomic entropy in Section **3**. Further we devote Section **4** to discuss the entanglement between the subsystem and sudden death. Finally we give our conclusion in Section **5**

2. Analytical solution

In this section we obtain the solution of the wavefunction from which we are able to construct the density matrix. To achieve this goal we firstly introduce the canonical transformation

$$\hat{a} = \hat{b}\cosh\zeta - \hat{b}^{\dagger}\sinh\zeta, \quad \hat{a}^{\dagger} = \hat{b}^{\dagger}\cosh\zeta - \hat{b}\sinh\zeta, \quad (3)$$

where $\zeta = \frac{1}{2}\tanh^{-1}(2\lambda_3/\omega)$ and the operators $\hat{b}(\hat{b}^{\dagger})$ are annihilation and creation operators and have the same meaning as the operators $\hat{a}(\hat{a}^{\dagger})$.. If we now substitute Eq. (3) into the Hamiltonian (1), then after a straightforward calculation we obtain the transformed Hamiltonian in the form [23–26]

$$\frac{\hat{H}}{\hbar} = \Omega\hat{n} + \sum_{j=1}^{2}\left(\frac{\Omega j}{2}\hat{S}_z^{(j)} + i\lambda_j\ \ (\hat{b}^{\dagger 2}\hat{S}_-^{(j)} - \hat{b}^2\hat{S}_+^{(j)})\right), \quad (4)$$

where we have considered both atoms to be identical. Note that, to derive the above Hamiltonian, we have applied the rotating–wave approximation with respect to the new operators $\hat{b}$ and $\hat{b}^{\dagger}$, not to the original (physical) operators $\hat{a}$ and $\hat{a}^{\dagger}$. In this case and as a result of applying the canonical transformation (3), the field frequency ω changes to $\Omega = \sqrt{\omega^2 - 4\lambda_3^2}$. Using the equations of motion in the Heisenberg picture the equations for the dynamics of the operators $\hat{n} = \hat{b}^{\dagger}\hat{b}$ and $\hat{S}_z^{(j)}$, $(j = 1, 2)$ can be written as

$$\frac{d\hat{n}}{dt} = 2\sum_{j=1}^{2}\lambda_j(\hat{b}^{\dagger 2}\hat{S}_-^{(j)} + \hat{b}^2\hat{S}_+^{(j)}),$$
$$\frac{d\hat{S}_z^{(1)}}{dt} = -2\lambda_1(\hat{b}^{\dagger 2}\hat{S}_-^{(1)} + \hat{b}^2\hat{S}_+^{(1)}), \quad \frac{d\hat{S}_z^{(2)}}{dt} = -2\lambda_2(\hat{b}^{\dagger 2}\hat{S}_-^{(2)} + \hat{b}^2\hat{S}_+^{(2)}), \quad (5)$$

from which we can show that

$$\hat{N} = \hat{n} + \sum_{j=1}^{2}\hat{S}_z^{(j)}, \quad (6)$$

where $\hat{N}$ is a constant operator. Using this fact the Hamiltonian model (4) can be written in the form

$$\frac{\hat{H}}{\hbar} = \Omega\hat{N} + \hat{C}, \quad (7)$$

where $\hat{C}$ is also a constant operator given by

$$\hat{C} = \sum_{j=1}^{2}\left(\frac{\Delta_j}{2}\hat{S}_z^{(j)} + i\lambda_j\ \ (\hat{b}^{\dagger 2}\hat{S}_-^{(j)} - \hat{b}^2\hat{S}_+^{(j)})\right). \quad (8)$$

In the above equation the quantity Δ_j is the detuning parameter defined by

$$\Delta_j = \Omega_j - 2\Omega, \quad j = 1, 2 \quad (9)$$

It is an easy task to show that the operators $\hat{N}$ and $\hat{C}$ commute and consequently each of them commutes with the Hamiltonian $\hat{H}$. Since the Hamiltonian is a constant of motion, the operator $\hat{C}$ is also a constant of motion. We now assume that the field is in the coherent state and the atoms are initially in pure atomic states. We further assume that the cavity field is initially prepared in a single–mode thermal state

$$\hat{\rho}_f(0) = \sum_{n=0}^{\infty}P_n|n\rangle\langle n|, \quad P_n = \frac{m^n}{(1+m)^n}, \quad (10)$$

where $m = [\exp(\hbar\omega/\kappa_\beta T) - 1]^{-1}$ with κ_β being Boltzmann's constant and T the temperature. The number m in the above equation is the mean photon number of the cavity field for thermal equilibrium at a certain temperature T. The density operator for the total system at the initial moment is given by

$$\rho_{aaf}(0) = \rho_f(0) \otimes \rho_{aa}(0). \quad (11)$$

Therefore at time $t > 0$ the joint density operator for the atom–field system takes the form

$$\rho_{aaf}(t) = \sum_{n=0}^{\infty}P_n(|R_1(n,t)|^2|n, ++ >< n, ++| + |R_2(n,t)|^2|n+2, +- >< n+2, +-| + R_2(n,t)R_3^*(n,t)|n+2, +- >< n+2, -+| + R_2^*(n,t)R_3(n,t)|n+2, -+ >< n+2, +-| + |R_3(n,t)|^2|n+2, -+ >< n+2, -+| + |R_4(n,t)|^2|n+4, -- >< n+4, --|), \quad (12)$$

where

$$R_j(n,t) = \sum_{i=1}^{4}A_{ij}(n,t) \quad (13)$$

and we have defined

$$A_{11}(n,t) = c_1\left[\frac{(\mu_1(n))^2 - 2\nu_1^2(n)(1 - \cos\mu_1(n)t)}{(\mu_1(n))^2}\right],$$
$$A_{12}(n,t) = -c_2\left[\frac{\Delta\ \ \nu_1(n)(1 - \cos\mu_1(n)t)}{(\mu_1(n))^2} + i\frac{\nu_1(n)\sin\mu_1(n)t}{\mu_1(n)}\right],$$
$$A_{13}(n,t) = -c_3\left[\frac{\Delta\ \ \nu_1(n)(1 - \cos\mu_1(n)t)}{(\mu_1(n))^2} - i\frac{\nu_1(n)\sin\mu_1(n)t}{\mu_1(n)}\right],$$