



# Medical image registration in fractional Fourier transform domain

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## ABSTRACT

The fractional Fourier transform as a powerful signal analysis and processing technique, the studies on using it for image registration are very rewarding and promising. This paper focuses on the medical image registration in the domain of fractional Fourier transform. The Powell algorithm is implemented to optimize the parameters by evaluating the minimum MSE between the magnitude of images. Two kinds of fractional Fourier transform are used to construct the algorithms. And they are tested on two sets of medica images and the experimental results were compared and analyzed.

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## 1. Introduction

Image registration is an important problem and a fundamental task in image processing technique because it provides an important aid for clinicians in diagnosis. Generally, image registration methods can be loosely divided into two classes: spatial domain approach and frequency domain approach. The phase correlation method is one of frequency domain approach, reference [1] was extended to determine rotation and scaling differences between two images by first converting the images to log-polar coordinates. This method is characterized by its insensitivity to translation, rotation, scale, and noise as well as by its low computational cost. However, in such cases, because of edge effects of Fourier transform and interpolation errors introduced by rotation and log-polar transform, a simple delta function will not be obtained, which will reduce the performance of the phase correlation method. To overcome the edge effects, a window function was added during the Fourier transform and the zero padded and gain function was used [2]. For triumph over the interpolation errors, authors in [3] introduced the pseudo-polar and pseudo-log-polar to replace the log-polar coordinates. Nevertheless, simulation show that the phase correlation method is highly desired toward the operator detail. It is difficult to obtain an accurate registration with this method, even using the window function, high pass filter and other methods, frequency aliasing cannot be absolutely eliminated.

The fractional Fourier transformation is an extension of the Fourier transform. It can be interpreted as a rotation by an angle  $\alpha$  in the time-frequency plane. The fractional Fourier transformation has been found to have many applications in the signal processing. It has many applications in image compression [4], watermarking [5], image encryption [6], pattern recognition [7], and image restoration [8,9]. Therefore, applications of the FrFT have been studied mostly in the areas of image processing. Currently there are few studies of the application in image registration. Sharma and Joshi proposed the use of FrFT in IR problem [10]. However, data is still scarce, we need to provide more calculation and description, further studies are still necessary. In this paper, we undertake to study image registration in the domains of two kinds of fractional Fourier transform.

The paper is organized as follows. In Section 2, review of the fractional Fourier transform are described. In Section 3, similarity measures and optimization algorithm for registration is displayed in detail. The results of simulation experiments are discussed in Section 4 and some words of conclusion make up Section 5.

## 2. The fractional Fourier transform

Up until now, the fractional Fourier transform has been digitally computed using a variety of approaches. In this paper two types FrFt techniques are compared and the registration results are presented, they are:

- (1) Discrete fractional Fourier transform [11].
- (2) Shin's fractionalization of Fourier transform [12].

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2.1. The discrete fractional Fourier transform

In literature [11] two digital algorithms of FrFT was given by Ozaktas et al. For ease of reference, we abbreviate it as DFRFT.

If  $F[f(x)]$  denote the Fourier transform of  $f(x)$ , the  $\alpha$ -th-order fractional Fourier transform is denoted  $F^\alpha[f(x)]$ . The DFRFT procedure can be represented as

$$f_\alpha = F^\alpha f, \tag{2.1}$$

$$F^\alpha = D \Lambda H_{lp} \Lambda J. \tag{2.2}$$

$D$  and  $J$  are matrices representing the decimation and interpolation operations.  $\Lambda$  is a diagonal matrix that corresponds to chirp multiplication, and  $H_{lp}$  corresponds to the convolution operation. The  $F^\alpha$  allows us to obtain the samples of the original function, which is the basic requirement for the definition of the discrete fractional Fourier transform matrix.

Overall, in the first step the function  $f(x)$  multiply a chirp function. The next step is to convolve  $g(x)$  with a chirp function. Then the last step, we obtain the samples of  $f_\alpha(x)$  spaced at  $1/2\Delta x$ .  $f_\alpha$  can write

$$f_\alpha(x) = \exp[-i\pi x^2 \tan(\phi/2)] g'(x), \tag{2.3}$$

$$g'(x) = A_\phi \int_{-\infty}^{\infty} \exp[i\pi\beta(x-x')^2] g(x') dx', \tag{2.4}$$

$$g(x) = \exp[-i\pi x^2 \tan(\phi/2)] f(x), \tag{2.5}$$

where  $\beta = \csc\phi$  and  $\phi = \alpha\pi/2$ .

2.2. The fractionalization of Fourier transform

Shih defined a fractional-order Fourier transform which is based on ordinary Fourier transform [12]. Considered the integer-order Fourier transform is a periodic operation. For the integer-order Fourier transform, there are four different functions,

$$\begin{aligned} F^1[f_0(x)] &= f_1(x) = g(x), \\ F^2[f_0(x)] &= F^1[f_1(x)] = f_2(x), \\ F^3[f_0(x)] &= F^1[f_2(x)] = f_3(x), \\ F^4[f_0(x)] &= F^1[f_3(x)] = f_0(x). \end{aligned} \tag{2.6}$$

where  $g(x)$  is the Fourier transform of the function  $f_0(x)$ . For the integer-order Fourier transform and we have  $f_3(x) = f_1(-x)$  and  $f_2(x) = f_0(-x)$ . These four functions, considered as the bases of the operation, and any fractional-order Fourier transform is a weighted combination of these four functions,

$$F^\alpha[f(x)] = A_0^\alpha f_0(x) + A_1^\alpha f_1(x) + A_2^\alpha f_2(x) + A_3^\alpha f_3(x), \tag{2.7}$$

The fractional-order Fourier transform can be simplified as

$$F^\alpha[f(x)] = \sum_{l=0}^3 f_l(x) \sum_{k=0}^3 \exp(-2\pi i k(\alpha - l)). \tag{2.8}$$

**Table 1**  
Range of the order  $\alpha$  accurately registration.

FrFT type	$\alpha$ Fig. 1 (Chest radiograph)			Fig. 2 (MRI of head)		
	(b) with (c)	(b) with (d)	(b) with (e)	(a) with (b)	(a) with (c)	(b) with (e)
Shin's	0–0.8, 1	0–1	0–0.97	0.6–1	0–1	Null
DFRFT	0–0.2, 0.8, 1	0–1	0–0.2, 0.6–0.9	0.1, 0.9, 1	0–1	0.1

3. The similarity measures and optimization algorithm

3.1. Registration as an optimization problem

The mean squared error (MSE) is measure of difference between two images and defined as follows:

$$MSE = \frac{1}{MN} \sum_{m=1}^M \sum_{n=1}^N (A(m, n) - B(m, n))^2, \tag{3.1}$$

where  $A(m, n)$  and  $B(m, n)$  represent two images have the same size of  $M \times N$  pixels. In this paper, the registration problem is seen as a searching of parameters of the transformation model that minimizes a cost function (MSE) in the fractional Fourier domain.

3.2. The Powell algorithm

The Powell algorithm has second-order convergence, and hence near a stationary value it converges more quickly [13]. Transformation of image can be represented by three parameters, they are translation on  $x$ -axis, translation on  $y$ -axis and rotation parameter. The Powell algorithm is used to optimize the three parameters.

**Step 1:** Let  $\vec{x}_0 = [0, 0, 0, 0]$  be an initial guess point. Let  $\vec{E}_n = \{0, \dots, 1_n, 0, \dots, 0\}$  for  $n = 1, 2, 3, 4$  be the set of standard base vectors. Initialize the vectors  $\vec{d}_n = \vec{E}_n$  and use their transpose  $\vec{d}_n^T$  to form the columns of the matrix  $D$  as follows:

$$D = [\vec{d}_1^T, \vec{d}_2^T, \vec{d}_3^T, \vec{d}_4^T],$$

$D$  is 4 linear independent directions, where  $n$  is 4-dimensional space because we will optimize 4 parameters. Initialize the counter  $k = 1$ .

**Step 2:** Set  $x^{(k,0)} = x^{(k-1)}$ , from the  $x^{(k,0)}$  to start, along the directions  $d^{(k,n)}$  to find the value of  $\lambda_n$  that minimizes  $f(x^{n,0} + \lambda_n d)$ , and obtained the points

$$x^{(k,1)}, x^{(k,2)}, \dots, x^{(k,n)}.$$

then start from the  $x^{(k,n)}$ , along the direction

$$d^{(k,n+1)} = x^{(k,n)} - x^{(k,0)},$$

to do one dimension search, and obtained point  $x^{(k)}$ .

**Step 3:** Stopping the algorithm while  $\|x^{(k)} - x^{(k-1)}\| < \epsilon$  are satisfied. Else let  $\Delta f = \max(f_{(k,n-1)} - f_{(k,n)})$ ,  $\text{temp} = \sqrt{f_{(k+1,0)} - f_{(k,0)}/\Delta f}$ , do one dimension search along the directions  $x^{(k+1,0)} - x^{(k,0)}$  to find the value of  $\lambda$  that minimizes  $f(x^{(k+1,0)} + \lambda)$ , if the  $|\lambda| \leq \text{temp}$  are satisfied, then set of direction vectors is left unchanged. If  $|\lambda| \geq \text{temp}$  then change the direction vectors  $d_n = d_{n+1}$ . Repeat Step1 through Step2 until convergence is achieved.

4. Experiment results and discussion

This section presents the experiment results. The registration is tested on two sets images. Fig. 1, below, shows the first set, in DICOM format. Fig. 1(a) is a 1760 × 1760 chest radiograph. In order to reduce test time, we cropped the image (a) to 318 × 318 as seen in Fig. 1(b). Fig. 2 shows second set, which are TIF format images. The 8-bit 256 × 256 MRI images of human head used for the experiments are shown in Fig. 2.

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