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Theoretical analysis of Yb^{3+}-doped double-clad fiber lasers using a new analytical method

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ABSTRACT

The approximate analytical solutions to the threshold pump power and output laser power of Yb^{3+}-doped double-clad fiber lasers have been deduced based on the rate equations with the scattering losses of pump and laser signal. In the rate equations, the relation between the attenuation of the pump light and the population density on the upper level is considered. The characteristics of Yb^{3+}-doped double-clad fiber lasers have been investigated.

1. Introduction

In recent years, rare earth-doped fiber lasers have drawn considerable attentions and made large progress experimentally [1–4] and theoretically [5–14]. Among these fiber lasers double-clad fiber lasers (DCFLs) have been actively studied. It is because of their characteristics including high output power, high conversion efficiency, easy heat extraction, excellent beam quality, large ration of surface area to volume, etc. Based on the rate equations for linear-cavity DCFL, Kelson and Hardy [7,10] have deduced the approximate analytic expressions of output power, lasing threshold, slope efficiency, the optimal fiber length and reflectivity of output mirror without the scattering loss. Xiao et al. [8] have deduced the approximate analytic solutions with scattering loss under the condition of the laser being strongly pumped, and shown three different pump modes. Kim et al. [9] and Peng et al. [11] have respectively investigated the DCFLs by numerical simulations based on the rate equations. However, one usually needs to guess the set of initial values in the numerical simulations, the results will be unstable if the initial values are not properly chosen. Luo et al. [13] and Liao et al. [14] have deduced the approximate analytic solution to the Yb^{3+}-doped DCFL using different algorithms, respectively. However, in these two works, the attenuation coefficient of the pump light along the fiber has been approximately regarded as a constant, that is the attenuation of the pump light have no relation with the change of the population density on the upper level. In this paper, it is considered that the attenuation coefficient of the pump light depends on the population density of the upper level. The characteristics of the DCFL have been investigated based on the rate equations with scattering losses.

2. Model and rate equations

The schematic diagram of a DCFL with forward pumping is shown in Fig. 1. The laser cavity is formed by a high reflectivity mirror (M_1) and a lower feedback mirror (M_2). Generally, the reflectivity of M_1 is close to 1, and that of M_2 is lower. When the laser is running in the steady state, there are forward signal laser $S^+(z)$ propagating along the fiber (from $z=0$ to $z=L$, i.e. positive z-direction) and backward signal laser $S^-(z)$ propagating in opposite direction (from $z=L$ to $z=0$, i.e. negative z-direction). The behavior of a linear-cavity Yb^{3+}-doped DCFL as shown in Fig. 1 running in the steady state is governed by rate equations [7,8]

$$\frac{dF(z)}{dz} = \Gamma_p[\sigma_{ep}N_2(z) - \sigma_{ap}N_1(z)]F(z) - \alpha_p F(z) \quad (1)$$

$$\pm\frac{dS^{\pm}(z)}{dz} = \Gamma_s[\sigma_{es}N_2(z) - \sigma_{as}N_1(z)]S^{\pm}(z) - \alpha_s S^{\pm}(z) \quad (2)$$

$$\frac{N_2(z)}{\tau} = u[\sigma_{ap}N_1(z) - \sigma_{ep}N_2(z)]F(z) + u[\sigma_{as}N_1(z) - \sigma_{es}N_2(z)] \times [S^+(z) + S^-(z)] \quad (3)$$

$$N = N_1(z) + N_2(z) \quad (4)$$

where $F(z)$ stands for the pump photon density (with the traveling direction of positive z-direction when the fiber laser is one-side pumped), $S^{\pm}(z)$ represent the laser photon densities propagating

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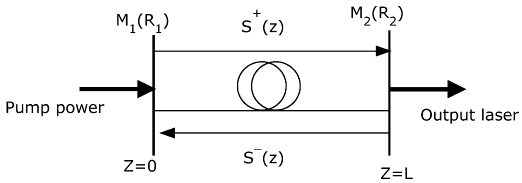

Fig. 1. Schematic diagram of Yb^{3+}-doped DCFL.

along the positive and negative z-directions, respectively, $N_1(z)$ and $N_2(z)$ are the population densities on the upper and lower energy level, respectively. N is the dopant concentration, τ is the relaxation time of an atom from the upper level to the lower level, u is the light speed inside the fiber, σ_a and σ_e are the absorption and emission cross-sections, respectively, Γ is the filling factor, α is the scattering loss coefficient, and the subscripts "s" and "p" indicate the laser and pump waves, respectively. From (1), it can be seen that the attenuation of the pump light depends on the variation of the population density of the upper level (N_2). The boundary conditions are

$$S^+(0) = S^-(0)R_1 \tag{5}$$

$$S^-(L) = S^+(L)R_2 \tag{6}$$

where L is the fiber length, R_1 and R_2 are the reflectivities of the laser mirrors of M_1 and M_2, respectively. From (2) it can be verified the $S^+(z)S^-(z)$ is independent on z, given that

$$S^+(z)S^-(z) = C^2 \tag{7}$$

where C is a constant. Thus (5) and (6) may be cast as

$$S^+(0) = Cr_1 \quad S^+(L) = \frac{C}{r_2} \tag{8}$$

$$S^-(0) = \frac{C}{r_1} \quad S^-(L) = Cr_2 \tag{9}$$

where r_1 and r_2 represent ${R_1}^{1/2}$ and ${R_2}^{1/2}$, respectively. From (2) and (4), one has

$$\ln\frac{S^+(L)}{S^+(0)} = \Gamma_s(\sigma_{as}+\sigma_{es})\int_0^L N_2 dz - \Gamma_s\sigma_{as}NL - \alpha_s L \tag{10}$$

Taking into consideration of (8) and (9), we have

$$\ln\frac{1}{r_1 r_2} = [\Gamma_s(\sigma_{as}+\sigma_{es})\eta - \Gamma_s\sigma_{as}N - \alpha_s]L \tag{11}$$

where

$$\eta = \frac{\int_0^L N_2 dz}{L} \tag{12}$$

Thus, from (11) and (12), one has

$$\eta = \frac{(1/L)\ln(1/r_1 r_2) + \alpha_s + \Gamma_s\sigma_{as}N}{\Gamma_s(\sigma_{as}+\sigma_{es})} \tag{13}$$

η represents the averaged population density on the upper level. It can be seen that η has no relation with the pump power. η depends on the parameters on the right-hand side of (13).

Considering (1) and (12), it can be deduced that

$$F(L) = F(0)\exp\{[\Gamma_p(\sigma_{ap}+\sigma_{ep})\eta - \Gamma_p\sigma_{ap}N - \alpha_p]L\} \tag{14}$$

$F(L)$ denotes the pump photon density at $z = L$. $F(0)$ denotes the pump photon density at $z = 0$. (14) indicates that the input pump photon density decreases along the fiber length. At $z = 0$, laser photon density can be expressed as $S(0) = Cr_1 + C/r_1$. At $z = L$, laser photon density can be expressed as $S(L) = Cr_2 + C/r_2$. Generally, $r_1 > r_2$, it can be seen that $S(L) > S(0)$, that is the pump photon density decreases while the laser photon density increases along the fiber length. It is known that the increase of the laser photon density results from the consuming of the population density of the upper level. Then, it is verified that $N_2(z)$ will decrease from $z = 0$ to $z = L$, i.e. $N_2(0) > N_2(L)$.

2.1. Threshold conditions

Under threshold condition, the laser photon density can be neglected, and (3) yields

$$\frac{N_2(z)}{\tau} = u[\sigma_{ap}N_1(z) - \sigma_{ep}N_2(z)]F(z) \tag{15}$$

From (1) and (3), one has

$$\frac{N_2(z)}{u\tau} = -\frac{dF(z)}{\Gamma_p dz} - \frac{\alpha_p F(z)}{\Gamma_p} \tag{16}$$

Direct integration from 0 to L on (16) leads to

$$\frac{\int_0^L N_2(z)dz}{u\tau} = -\frac{1}{\Gamma_p}\int_0^L dF(z) - \frac{\alpha_p}{\Gamma_p}\int_0^L F(z)dz \tag{17}$$

Because the value of $F(z)$ decreases monotonously along the fiber length, here, an approximation is used as follows

$$\frac{\int_0^L F(z)dz}{L} \approx \frac{F(0)+F(L)}{2} \tag{18}$$

The left-hand side of (18) indicates the averaged pump photon density along the whole fiber length. Inserting (18) into (17) and considering (12) and (14), one can deduce that

$$F(0)_{\text{th}} = \frac{\eta L}{u\tau\{-((2+\alpha_p L)/2\Gamma_p)\exp\{[\Gamma_p(\sigma_{ap}+\sigma_{ep})\eta - \Gamma_p\sigma_{ap}N - \alpha_p]L\} + ((2-\alpha_p L)/2\Gamma_p)\}} \tag{19}$$

Here, $F(0)_{\text{th}}$ stands for the threshold pump photon density. The relation between the threshold pump photon density and its related power can be expressed as

$$P_{\text{th}} = \frac{h\upsilon_p A u F(0)_{\text{th}}}{\Gamma_p} \tag{20}$$

where υ_p is the frequency of the pump and A is the cross-sectional area of the fiber core.

2.2. Strongly pumped laser

When the pump power is higher than the threshold, the laser photon density cannot be neglected. Considering (1)–(3), one has

$$\frac{N_2(z)}{u\tau} = -\frac{dF(z)}{\Gamma_p dz} - \frac{\alpha_p F(z)}{\Gamma_p} - \frac{\alpha_s S^+(z)}{\Gamma_s} - \frac{dS^+(z)}{\Gamma_s dz} - \frac{\alpha_s S^-(z)}{\Gamma_s} + \frac{dS^-(z)}{\Gamma_s dz} \tag{21}$$

From (21), one has

$$\frac{\int_0^L N_2(z)dz}{u\tau} = -\frac{\int_0^L dF(z)}{\Gamma_p} - \frac{\alpha_p\int_0^L F(z)dz}{\Gamma_p} - \frac{\alpha_s\int_0^L S^+(z)dz}{\Gamma_s} - \frac{\int_0^L dS^+(z)}{\Gamma_s} - \frac{\alpha_s\int_0^L S^-(z)dz}{\Gamma_s} + \frac{\int_0^L dS^-(z)}{\Gamma_s} \tag{22}$$

It is known that the value of $S^+(z)$ (or $S^-(z)$) increases (or decreases) monotonously from $z = 0$ to $z = L$, an approximation is used as follows

$$\frac{\int_0^L S^\pm(z)dz}{L} \approx \frac{S^\pm(0)+S^\pm(L)}{2} \tag{23}$$