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Two wave mixing in nonlinear Kerr medium and its dependence on photoconductivity and dielectric properties of the medium

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ABSTRACT

Two-wave mixing spectroscopy in Kerr media is shown to be a powerful technique to quantify the strong enhancement of the phase shift and gain in signal beam. The phase shift and gain in signal beam are important parameters for unidirectional ring resonator, an analytical solution for the two wave mixing gain and phase shift in signal in a nonlinear Kerr medium is presented. We have considered only non-degenerate two wave mixing by introducing the grating decay time in terms of photoconductivity and dielectric constant of the Kerr media. Effect of the oscillation frequency shift and angle between the two interfering beams on phase shift and gain of the signal beam has been studied. We have also studied the influence of photoconductivity and dielectric constant of the Kerr media on phase shift and gain of the signal beam in details.

1. Introduction

Recent years have shown increased interest from different experimental and theoretical groups in the study of the propagation of electromagnetic wave in the Kerr media [1–6]. The optical Kerr effect manifests itself temporally as self-phase modulation, a self-induced phase and frequency-shift of a pulse of light as it travels through a medium [7–10]. Two-wave mixing in Kerr media is a physical process which takes advantage of the nonlinear response of some materials to the illumination of electromagnetic radiation. The phase shift and gain is an important parameter for unidirectional ring resonator [11,12]. In order to describe the wave mixing in Kerr media, let us consider the interference pattern formed by two laser beams in a nonlinear medium. Such a pattern is characterized by a spatial variation (usually periodic) of the intensity. If the medium responds nonlinearly, then an index variation is induced in the medium. The process of forming an index variation pattern inside nonlinear medium using two-beam interference is similar to that of hologram formation. Such an index variation pattern is often periodic and is called a volume index grating [13,14]. When the two waves propagate through the grating induced by them, they undergo Bragg scattering [15]. One beam scatters into the other and vice versa. Such scatterings are reminiscent of the read-out process in holography [16].

The concept of using moving gratings in local media for energy coupling was first proposed in 1973 by a group of Soviet scientists [17,18]. It was recognized that a spatial phase shift between the index grating and the light intensity pattern can be induced by moving the grating relative to the medium. Such a spatial phase shift is a result of the inertia (temporal) of the hologram formation process and leads to a nonreciprocal energy transfer. If the formation time of the hologram is finite, a spatial phase shift occurs when the intensity pattern is moving relative to the medium. In addition to the phase shift, such a motion also leads to a decrease in the depth of modulation of the induced index grating. Several possibilities of achieving such a spatial phase shift have been proposed. These include moving the medium itself relative to a thermally induced grating [17], using the Lorentz force to move free carrier grating in a semiconducting medium [18], and non-degenerate two-wave mixing in which a frequency shift between the beams results in a moving grating [18,19]. It is important to note that a temporal phase shift itself is not enough for energy coupling. The induced index grating must be physically shifted in space relative to the intensity pattern in order to achieve energy coupling. It is known that the Kerr effect in gases or fluids is a local effect. In media with local response [13,14], there is no steady-state transfer of energy between two lasers of the same frequency. In what follows, we will show that nonlocal response can be induced by moving the Kerr medium relative to the beams. Such an induced nonlocal response is only possible when the material response time τ is finite.

In the present paper, we have focused on the coupling of two electromagnetic waves inside the Kerr medium and discuss the phase shift and gain of the signal beam. Since the phase shift and gain in the signal beam are important parameters for the unidirectional ring resonator [11,12]. In this paper, we have introduced the concept of the grating decay time in case of non-degenerate two

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wave mixing, in terms of photoconductivity and dielectric properties of the Kerr media for the first time to the best of our knowledge, which are not considered in the earlier published articles. The dependence of wave mixing gain and phase shift of signal beam on oscillation frequency shift and angle between the interfering beams have been studied in details. Moreover, effects of the photoconductivity, dielectric constant of the Kerr media on phase-shift and gain of the signal beam have been explored in details.

2. Mathematical description

When the electromagnetic waves progress in the media possessing a strong Kerr effect a number of interesting phenomena, e.g., self-phase modulation, mode-locking and self-focusing manifest at high incident beam power. The dependence of the index of refraction on the electric field in the Kerr media is described by

$$n = n_0 + n_2 E_{av}^2 \tag{1}$$

where n_0 is the index of refraction, n_2 is the Kerr coefficient and E_{av}^2 is the average of the varying electric field. In the case of degenerate two wave mixing process, the time average of the varying electric field, is given by

$$E_{av}^2 = E_0^2[1 + \cos(\vec{K}.\vec{r})] \tag{2}$$

where $E_0 = A_p = A_s$ are amplitudes. Now using Eqs. (1) and (2), we have

$$n = n_0 + n_2 E_0^2[1 + \cos(\vec{K}.\vec{r})] \tag{3}$$

Comparing (2) with (3) we note that the response is local and there can be no energy coupling, even if n_2 is complex. Inside the Kerr medium, the two waves form an interference pattern which corresponds to a spatially periodic variation of the time-averaged field (E_{av}^2). In a Kerr medium, such a periodic intensity produces a volume grating. The formulation of two-wave mixing in Kerr media is very similar to that of the holographic two-wave coupling in photorefractive crystals [20–22]. However, there exists a fundamental difference between these two types of two wave mixing. In photorefractive media, the index modulation is proportional to the contrast of the interference fringes, whereas in Kerr media the index modulation is directly proportional to the field strength. Thus, in Kerr media the coupling strength is proportional to the beam intensities, whereas in photorefractive media the coupling strength is determined by the ratio of beam intensities.

First, we consider the situation that a pulse, which consists of two frequency components ω_p and ω_s interacts with a nonlinear Kerr medium. The electric fields E_p and E_s of the two coupling beams can be written as:

$$E_p = A_p \exp[j(\omega_p t - \vec{k}_p.\vec{r})] \tag{4}$$

$$E_s = A_s \exp[j(\omega_s t - \vec{k}_s.\vec{r})] \tag{5}$$

where A_p and A_s are the amplitudes and are taken as functions of z only for a steady state situation and $\rightarrow k_p$ and $\vec{k}_s$ are the wave vectors of the pump and signal beams, respectively, $j = \sqrt{-1}$. Here, t and r are the time and space coordinates, respectively. For simplicity we assume that both waves are s-polarized and the medium is isotropic. The z-axis is taken normal to the surface to the medium.

When the two optical coherent beams progress in the Kerr medium an interference pattern is generated. Such a pattern is moving with a certain velocity if $\omega_p \neq \omega_s$. The electric field (i.e., total electric field) of the interference pattern is given by:

$$E = E_p + E_s \tag{6}$$

And averaging is taken over a time interval T in such a way that

$$\omega_p T \gg 1, \qquad \omega_s T \gg 1 \tag{7}$$

and

$$|\omega_s - \omega_p| T \ll 1 \tag{8}$$

Now using

$$E_{av}^2 = \frac{1}{2}\mathrm{Re}[E + E]$$

Using Eqs. (4)–(6), we obtain

$$E_{av}^2 = \frac{1}{2}\left\{|A_p|^2 + |A_s|^2 + A_p^* A_s\ e^{i(\Omega t - \vec{K}.\vec{r})} + A_p A_s^*\, e^{i(\Omega t - \vec{K}.\vec{r})}\right\} \tag{9}$$

where

$$\Omega = \omega_s - \omega_p \tag{10}$$

$$\vec{K} = \rightarrow k_s - \vec{k}_p \tag{11}$$

This interference pattern induces a volume index grating via the Kerr effect and for the sake of convenience we assume a scalar grating. In general, there is a spatial phase shift between the interference pattern and the volume index grating because of the time-varying nature of the pattern. Thus, one can write the index of refraction (1) including the fundamental components of the Kerr-induced grating in generalize form as

$$n = n_0 + \Delta n_0 + \frac{1}{2}\left\{n_2\ e^{i\Phi} A_p^* A_s\, e^{i(\Omega t - \vec{K}.\vec{r})} + c.c\right\} \tag{12}$$

where Δn_0 is a uniform change in index of refraction, Φ and n_2 are real quantity. Here Φ is the phase to which the index grating is spatially shifted (or temporally delayed) with respect to the interference pattern and both n_2 and Φ are functions of Ω. The phase shift Φ is finite it is due to the finite response of the material. In Eq. (12) $n_2\, e^{i\Phi}$ can be regarded as a complex Kerr coefficient and it is corresponds to a complex third-order nonlinear optical polarizability. The imaginary part of the third-order nonlinear optical polarizability is responsible for the stimulated Brillouin scattering and stimulated Raman scattering [23]. Thus, the complex Kerr coefficient induced due to moving gratings will responsible for the exchange in energy between the two interfering beams.

When the frequency of the two interfering waves are same (i.e., $\omega_s = \omega_p$) then the wave mixing is called degenerate two wave mixing, in this case a steady-state nonlinear response without phase shift is observed and which is described by (1). In the case of non-degenerate two wave mixing, the intensity fringe pattern, as described by (9), is moving with a constant speed $v = (\Omega/k) = (\Omega\Lambda/2\pi)$ in the nonlinear medium. In order to derive the steady-state value of the self-induced index change the finite response time of the medium is considered with respect to the displacement speed. Let the decay of index change be exponential, the steady-state index change can be written

$$\Delta n = \frac{1}{\tau} n_{20} \int_{-\infty}^{t} E_{av}^2(t')\ e^{(t'-t)/\tau}\, dt' \tag{13}$$

where n_{20} represent the change in the index of refraction for the degenerate two-wave mixing process in the nonlinear Kerr medium.

Now integration of Eq. (13) yields the following expression for $n_2 \exp(i\Phi)$

$$n_2\ \exp(i\Phi) = \frac{n_{20}}{1 + i\Omega\tau} \tag{14}$$

where

$$\Phi = \tan^{-1}\ \Omega\tau \tag{15}$$

and τ is the response time of the medium.

When the beams enter into the medium from the same side, i.e., at $z = 0$ the coupling is known as co-directional two-beam coupling. To investigate the coupling in Kerr media we substitute n (12) for