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Stability in a general coupled laser array

Liang Zhou*, Kailiang Duan

State Key Laboratory of Transient Optics and Photonics, Xi'an Institute of Optics and Precision Mechanics, Chinese Academy of Sciences, Xi'an 710119, China

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ABSTRACT

For the first time we have presented a method to investigate the stability of quiescent state in a coupled laser array. By introducing tiny perturbation, stability of quiescent state can be known by investigating eigenvalues of coefficient matrix of the corresponding linear differential equations. Only when real parts of the eigenvalues were all negative or zero, the corresponding quiescent state is stable. Two tapes of coupled laser array with loop and linear topological structure were studied respectively. It was found that there were innumerable quiescent states and they could be divided into several types based on phase relation. Some types were stable and others were unstable. Topological structure affects the stability of quiescent state in a coupled laser array.

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1. Introduction

Many interesting phenomena have been found in various coupled laser arrays and much attention has been paid to the investigation of coupled laser arrays in recent years. Winful and Rahman [1] investigated the possibility of chaos in a coupled laser array theoretically. Roy and Thornburg [2] observed the synchronization in a coupled laser array experimentally. Dai and Yin [3] developed a method for controlling chaos in a hybrid optical bistable system. Terry and Thornburg [4] studied the chaos of three Nd:YAG lasers oriented in a linear array. Jiang et al. [5] studied numerical synchronization and message transmission between two mutually coupling semiconductor lasers subject to identical unidirectional injections from an external cavity semiconductor. Zhou et al. [6] reported phase-locked state of a coupled laser array in a loop array experimentally.

All the above works focus mainly on the chaos or method to achieve phase-locked state in coupled laser array. Phase-locked state is also called as quiescent state. To our knowledge, however, stability of quiescent state in coupled laser array has not yet been reported. In many applications, such as coherent beam combination [6] and phased array radar [7], stability of quiescent state is important, because there are always various disturbances in practical use and only stable quiescent state can exist. Investigation of stability of quiescent state in coupled laser array can provide further insight into the nature of coupled laser array system. This is the motivation of this article.

2. Theory and methods

In this article, after elucidating the method of studying stability of quiescent states in a general chaotic coupled laser array, two coupled laser arrays, in loop and linear array respectively, are taken for examples. It is found that many kinds of quiescent states exist and some kinds are stable while others are unstable. It is pointed that quiescent state and its stability are related to the topological structure of the coupled laser array.

Quiescent state is the state in which the amplitudes and gains are constant and rates of phases are only optical frequency. To get a universal method of studying stability of quiescent states, we first consider an arbitrary coupled laser array. The topological structure of the arbitrary coupled laser array is illustrated in Fig. 1. For generality, these lasers can be fiber lasers or semiconductor lasers or Nd:YAG lasers, etc. It is supposed that each laser operates in the same longitudinal and transverse modes at the same optical frequency.

The evolutions of the electric field $E_j(t)$ of the guided mode and the gain $G_j(t)$ of the jth laser ($j = 1, 2, 3, \ldots, N$) are described by the following nonlinear differential equations [8,9]

$$\frac{dE_j}{dt} = \left[i\omega_0 + \frac{1}{\tau_c}(G_j - a_j)\right] E_j - \frac{1}{\tau_c} \sum_{n=1, n\neq j}^{N} \kappa_{nj} E_n \quad (1)$$

$$\frac{dG_j}{dt} = \frac{1}{\tau_f}(p_j - G_j - G_j|E_j|^2) \quad (2)$$

* Corresponding author at: No. 17 Xinxi Road, New Industrial Park, Xi'an Hi-Tech Industrial Development Zone, Xi'an, Shaanxi, China. Tel.: +86 132 42826531; fax: +86 029 88887603.
E-mail address: zql_zhouliang@126.com (L. Zhou).

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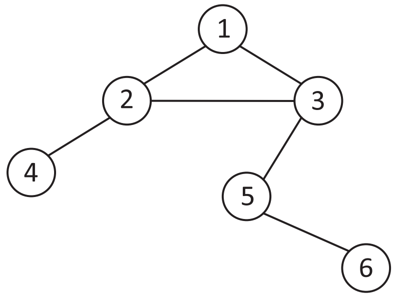

Fig. 1. Topological structure of an arbitrary coupled laser array. Circles represent lasers. Short lines represent the coupling relationship between lasers. j denotes jth laser, j = 1, 2, 3, 4, 5, 6.

where ω_0 is the optical frequency, p_j is the pump coefficient, α_j is the cavity loss coefficient, τ_c is the cavity round trip time, τ_f is the fluorescence time of the upper lasing level, κ_{nj} is the coupling strength between nth lasers and jth lasers, and N is the total number of lasers. By writing the electric field in forms of $E_j(t) = X_j(t)\exp[-i\phi j(t)]$ and $\phi_j(t) = \theta_j(t) + \omega_0 t$, where $\phi_j(t)$, $\theta_j(t)$ and $X_j(t)$ are real functions of the field phase and amplitude, respectively, Eqs. (1) and (2) can then be transformed into forms of

$$\frac{dX_j}{dt} = \frac{1}{\tau_c}(G_j - a_j)X_j - \frac{1}{\tau_c}\sum_{n=1, n\neq j}^{N} \kappa_{nj} X_n \cos(\theta_n - \theta_j) \tag{3}$$

$$\frac{d\phi_j}{dt} = -\sum_{n=1, n\neq j}^{N} \frac{\kappa_{nj} X_n}{\tau_c X_j}\sin(\theta_n - \theta_j) \tag{4}$$

$$\frac{dG_j}{dt} = \frac{1}{\tau_f}(p_j - G_j - G_j X_j^2) \tag{5}$$

Eqs. (3)–(5) are nonlinear differential equations that govern the evolutions of the intensity phases and gains of the coupled laser array. Quiescent states can be described by constant solution of Eqs. (3)–(5), which can denoted by $(X_{10}, \ldots, X_{N0}, \theta_{10}, \ldots, \theta_{N0}, G_{10}, \ldots, G_{N0})^T$.

Stable constant solution corresponds to stable quiescent states. In order to discuss stability of constant solutions, it is assumed that there are tiny perturbations of these quiescent states, denoted by $(\alpha_1, \ldots, \alpha_N, \eta_1, \ldots, \eta_N, \beta_1, \ldots, \beta_N)^T$. Substituting $X_j = X_{j0} + \alpha_j$, $\theta_j = \theta_{j0} + \eta_j$, $G_j = G_{j0} + \beta_j$ (j = 1, 2, 3, ..., N) into Eqs. (3)–(5), and abandoning higher order terms, we can get linear differential equations

$$\frac{d\boldsymbol{y}}{dt} = \boldsymbol{A}\boldsymbol{y} \tag{6}$$

$$\boldsymbol{y} = (\alpha_1, \ldots, \alpha_N, \eta_1, \ldots, \eta_N, \beta_1, \ldots, \beta_N)^T$$

$\boldsymbol{A}$ is a $3 \times N$ order square matrix. For convenience, $\boldsymbol{A}$ is written in the form of block matrix

$$\boldsymbol{A} = \begin{pmatrix} \boldsymbol{B} & \boldsymbol{C} & \boldsymbol{D} \\ \boldsymbol{E} & \boldsymbol{F} & \boldsymbol{K} \\ \boldsymbol{H} & \boldsymbol{I} & \boldsymbol{J} \end{pmatrix}$$

where

$$\boldsymbol{B} = \begin{pmatrix} B_{11} & \cdots & B_{1N} \\ \vdots & \ddots & \vdots \\ B_{N1} & \cdots & B_{NN} \end{pmatrix}, \qquad B_{mm} = \frac{G_{m0} - a_m}{\tau_c},$$

$$B_{mn} = \frac{-\kappa_{nm} \times \cos(\theta_n - \theta_m)}{\tau_c} \; (m \neq n)$$

$$\boldsymbol{C} = \begin{pmatrix} C_{11} & \cdots & C_{1N} \\ \vdots & \ddots & \vdots \\ C_{N1} & \cdots & C_{NN} \end{pmatrix}, \quad C_{mm} = \sum_{\substack{r=1 \\ r \neq m}}^{N} \frac{-\kappa_{rm} \times X_{r0} \times \sin(\theta_r - \theta_m)}{\tau_c},$$

$$C_{mn} = \frac{\kappa_{nm} \times X_{n0} \times \sin(\theta_n - \theta_m)}{\tau_c} \; (m \neq n)$$

$$\boldsymbol{D} = \begin{pmatrix} D_{11} & \cdots & D_{1N} \\ \vdots & \ddots & \vdots \\ D_{N1} & \cdots & D_{NN} \end{pmatrix}, \qquad D_{mm} = \frac{X_{m0}}{\tau_c}, \quad D_{mn} = 0 \; (m \neq n)$$

$$\boldsymbol{E} = \begin{pmatrix} E_{11} & \cdots & E_{1N} \\ \vdots & \ddots & \vdots \\ E_{N1} & \cdots & E_{NN} \end{pmatrix}, \quad E_{mm} = \sum_{\substack{r=1 \\ r \neq m}}^{N} \frac{\kappa_{rm} \times X_{r0} \times \sin(\theta_r - \theta_m)}{X_{m0}^2 \tau_c},$$

$$E_{mn} = \frac{-\kappa_{nm} \times \sin(\theta_n - \theta_m)}{X_{m0}\tau_c} \; (m \neq n)$$

$$\boldsymbol{F} = \begin{pmatrix} F_{11} & \cdots & F_{1N} \\ \vdots & \ddots & \vdots \\ F_{N1} & \cdots & F_{NN} \end{pmatrix}, \quad F_{mm} = \sum_{\substack{r=1 \\ r \neq m}}^{N} \frac{\kappa_{rm} \times X_{r0} \times \cos(\theta_r - \theta_m)}{X_{m0}\tau_c},$$

$$F_{mn} = \frac{-\kappa_{nm} \times \cos(\theta_n - \theta_m) \times X_{n0}}{X_{m0}\tau_c} \; (m \neq n)$$

$$\boldsymbol{K} = \begin{pmatrix} K_{11} & \cdots & K_{1N} \\ \vdots & \ddots & \vdots \\ K_{N1} & \cdots & K_{NN} \end{pmatrix}, \quad K_{mn} = 0$$

$$\boldsymbol{H} = \begin{pmatrix} H_{11} & \cdots & H_{1N} \\ \vdots & \ddots & \vdots \\ H_{N1} & \cdots & H_{NN} \end{pmatrix}, \qquad H_{mm} = \frac{-2 \times G_{m0} \times X_{m0}}{\tau_f},$$

$$H_{mn} = 0 \; (m \neq n)$$

$$\boldsymbol{I} = \begin{pmatrix} I_{11} & \cdots & I_{1N} \\ \vdots & \ddots & \vdots \\ I_{N1} & \cdots & I_{NN} \end{pmatrix}, \quad I_{mn} = 0$$

$$\boldsymbol{J} = \begin{pmatrix} J_{11} & \cdots & J_{1N} \\ \vdots & \ddots & \vdots \\ J_{N1} & \cdots & J_{NN} \end{pmatrix}, \quad J_{mm} = \frac{-x_{m0}^2 - 1}{\tau_f}, \quad J_{mn} = 0 \; (m \neq n)$$

$$(m, n = 1, \ldots, N)$$

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