Optik 123 (2012) 1659–1668

Contents lists available at SciVerse ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Radiometry of quasi-homogeneous sources and Wigner radiometry

Andrey V. Gitin *

Max-Born-Institut für Nichtlineare Optik und Kurzzeitspektroskopie, Max-Born-Str. 2a, D-12489 Berlin, Germany

ARTICLE INFO

Article history:
Received 13 April 2011
Accepted 29 August 2011

Keywords:
Radiometry
Vignetting
Eikonal theory
Isoplanar optical system
Quasi-homogeneous sources

ABSTRACT

The radiometry of quasi-homogeneous sources is derived from the theory of partial coherence by using properties of the Wigner definition function. Suggested is the linear-system approach to the process of forming an optical image in which the convolution operator is used twice. First to describe influence of aberrations of an optical system on the image quality. Second to describe influence of the radiation pattern of the source-object and the form and the size of the entrance pupil on the fall-off of the irradiance from centre of the image plane to its edge.

1. Introduction

Traditional radiometry is based on the *a priory* assumption that the source radiates according to the Lambert's law. It has a very wide field of applications: from the theory of blackbody radiation to the lighting engineering [1].

If this field of applications of radiometry is restricted to energy calculations of optical systems [2–5] then there are two approaches to extend the concept of the traditional radiance to the case of non-Lambertian planar sources. The first approach is based on the Hamilton optics and properties of the phase space [4–6] (so-called the Hamilton radiometry), the second one is based on the Fourier optics, the theory of partial coherence, and properties of the Wigner definition function (WDF) [7] (so-called the Wigner radiometry).

In the works [3,5,8,9] we suggested to use the radiometry quasi-homogeneous sources for energy calculations of "natural vignetting" in perfect optical systems. It is convenient because it allows using the operator of convolution for calculating the fall-off of the irradiance from centre of the image plane to its edge. In the articles [5,10], the radiometry of quasi-homogeneous sources was justified on the basis of the Hamilton radiometry.

In this article we will derive the basic relation of the radiometry quasi-homogeneous sources from the Wigner radiometry.

2. The Fourier optics

In scalar quasi-monochromatic approach, that is, neglecting polarisation and spectral structure of radiation, distribution of a wave field in the point (x,y,z) of the space can be described by the complex amplitude $U(x,y;z)$. Propagation of the scalar quasi-monochromatic field with wavelength λ_o in vacuum through an inhomogeneous optical medium with the refractive index distribution $n(x,y,z)$ is described by the Helmholtz equation [11–13]

$$\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}+k_o^2 n^2(x,y;z)U=0 \tag{1}$$

where $k_o \equiv 2\pi/\lambda_o$ is a wave number. The Helmholtz equation is the main equation of the wave optics.

* Corresponding author.
E-mail address: agitin@mbi-berlin.de

0030-4026/$ – see front matter © 2011 Elsevier GmbH. All rights reserved.
doi:10.1016/j.ijleo.2011.08.042

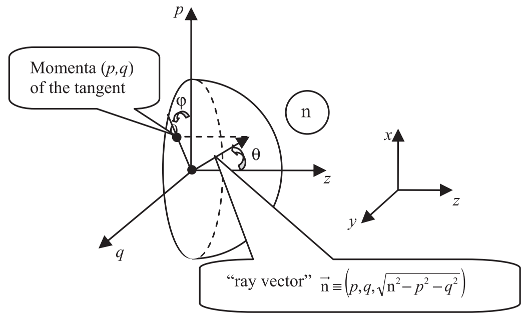

Fig. 1. Direction of the “ray vector” can be characterized by angles (θ,φ) of the spherical coordinate system ($\varphi \in (0,2\pi)$, $\theta \in (0,\pi/2)$), or by momenta (p,q).

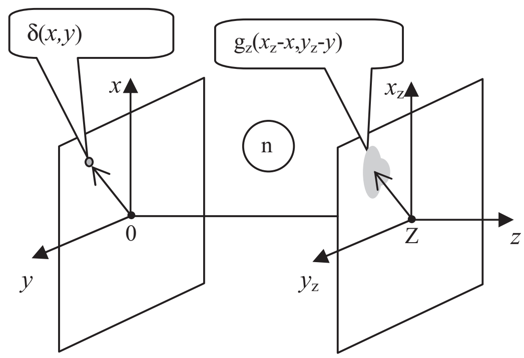

Fig. 2. A layer of a free space of thickness Z as a linear shift-invariant system.

A plane wave travelling through a homogeneous optical medium with refractive index n in a direction specified by the vector $(p, q, \sqrt{n^2 - p^2 - q^2})$ (see Fig. 1) is described by the equation

$$\exp\left[ik_o(xp + yq + z\sqrt{n^2 - p^2 - q^2})\right] \tag{2}$$

Note that in the Hamilton optics the direction cosines of the wave vector p, q are called momenta.

Let us consider the plane-parallel layer of this optically homogeneous medium bordered by a parallel input plane $(x,y)=(x,y,0)$ and an output plane $(x_z,y_z)=(x,y,Z)$ (Fig. 2). Note that, using the direction cosines p and q as two independent variables, we can describe the complex amplitude in the output plane, $U(x,y;Z)$, as a lineal superposition of plane waves with a weight function $\tilde{U}_{in}(p, q)$ (so-called a plane wave expansion) [11–13]:

$$U(x, y; Z) = \iint_{R^2} \tilde{U}_{in}(p, q)\exp\left[ik_o(xp + yq + Z\sqrt{n^2 - p^2 - q^2})\right] dpdq \tag{3}$$

This lineal superposition satisfies the Helmholtz Eq. (1) also.There are the direct and the inverse k_o-Fourier transformations [14]

$$\tilde{U}(p, q) = F_{\substack{x \to p \\ y \to q}} \left\{U(x, y)\right\} \equiv \iint U(x, y) \cdot \exp\left[-ik_o(xp + yq)\right] dxdy \tag{4a}$$

$$U(x, y) = F^{-1}_{\substack{p \to x \\ q \to y}} \left\{\tilde{U}(p, q)\right\} \equiv \iint \tilde{U}(p, q) \cdot \exp\left[ik_o(xp + yq)\right] dpdq \tag{4b}$$

Note that if $Z = 0$ the plane wave expansion (3) takes the form of the inverse k_o-Fourier transformation

$$U(x, y; 0) = F^{-1}_{\substack{p \to x \\ q \to y}} \left\{\tilde{U}_{in}(p, q)\right\} \tag{5}$$

Thus, using the direct k_o-Fourier transformation (4b) Eq. (3) can be rewritten as

$$\tilde{U}_{out}(p, q) = \tilde{U}_{in}(p, q) \cdot \exp\left[-ik_o Z \cdot H(p, q)\right] \tag{6}$$

Here

$$\tilde{U}_{in}(p, q) = F_{\substack{x \to p \\ y \to q}} \left\{U_{in}(x, y)\right\}$$