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Application of wavelet filtering techniques to Lau interferometric fringe analysis for measurement of small tilt angles

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ABSTRACT

This communication presents the results of investigation undertaken towards testing the viability of wavelet filtering techniques in analyzing the Lau interferometric fringes. Towards this, the applicability of Haar, Daubechies and Symlet wavelet based filtering schemes have been tested. Of these schemes, the Symlet based filtering scheme gives the best results in terms of improved signal to noise ratio. This scheme is then used to analyze Lau interferograms recorded in the experimental arrangement for tilt angle measurement. The processed interferograms yield improved accuracy and precision of measurement.

1. Introduction

Lau effect/Interferometry has been used extensively for applications in Science and Engineering. A detailed account of the theory and applications of Lau effect/interferometer has been provided by Patorski [1]. Detailed applications of this phenomenon for visualization of phase objects, decoding theta modulated information, image processing, etc. have been reported by the author. In recent times, the use of Lau effect/interferometer for measurement of focal length [2], temperature [3,4], refractive index [5], parallelism of the plate [6], degree of coherence [7], small tilt angle [8], etc. has been reported. It has also been used for fabrication of array illuminators [9], theta-encoder [10] and for realization of refractometer for testing the eye-sight [11]. The use of Lau interferometer for determination of the focal length of collimating lens [12], profiling of reflecting surface [13] and in vibration analysis [14] has also been demonstrated. Among various advantages offered by the Lau based interferometers, is the fact that they use inexpensive components. Since the technique uses the incoherent light, the coherent noise is minimized.

Lau interferogram like any other common interference fringe pattern contains the information about the wave fronts interfering at the detector plane. Hence, the analysis of this interferogram yields the information about the interfering wave fronts. In general, the measurement variable is related to the fringe spacing or the fringe inclination angle in the interferogram. However, accurate determination of this data is difficult to achieve because the directly recorded Lau interferogram has poor brightness and contrast. The grain shaped random noise is present over the interferogram. Over this, is superimposed the intensity and contrast variations due to light reaching the image plane after being scattered from the surface of various components. Also, the noise due to grating lines overlaps the interferogram. These factors limit the characteristics of the measurement systems involving Lau effect/interferometer. This fact has been reported in several studies [2,8,9] and hence is the limiting factor of the technique. To circumvent these problems, one of the approaches suggested has been the incorporation of phase shifting techniques into the Lau interferometric set-up. However, this involves incorporation of precise grating translation devices into the set-up. Besides adding to the cost, such measures cannot be fruitful in case of dynamic applications such as vibration measurement. In the present communication, we test the applicability of the wavelet based filtering techniques to de-noise the recorded Lau interferogram and use the technique for measurement of small tilt angle.

Several image de-noising algorithm have been proposed in the literature. These algorithms have been tested on variety of images such as normal pictures, moiré based fringes, digital speckle based fringes, tomographic images. Recently, in all these formats, there has been incorporation of wavelet filtering techniques because of inherent advantages in its use. Grace Chang et al. used spatially adaptive wavelet thresholding for denoising the standard test pictures [15,16]. Gdeisat et al. proposed the spatial fringe pattern demodulation using two dimensional continuous wavelet transform [17,18]. Iftekharuddin proposed the use of harmonic wavelet joint transform correlator for image noise reduction in noisy syn-

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thetic aperture (SAR) images [19]. Liu et al. combined wavelet based image processing to extract precise phase information in phase shifting moiré interferometry. The discrete wavelet transform has been used to determine in-plane strain by eliminating noise generated through the derivative calculation [20]. Chia-Ming et al. [21] have reported the use of wavelet transform to remove the noise embedded in Moiré interferogram recorded using digital AFM Moiré method. The use of second-order Daubechies wavelet function for image filtering has been reported. Authors demonstrated that the accuracy of the digital AFM Moiré method is greatly improved by incorporating wavelet filtering methods in phase-shifting interferometry. The method has been used for in-plane displacement, nano scale in-plane displacement and strain field measurement. Donoho and Johnstone [22,23] proposed a wavelet domain thresholding and shrinkage technique for noise reduction. This and related methods were used by Lang et al. for noise reduction in NMR spectra and synthetic aperture radar signals [24], and in astronomical data [25]. Excellent noise reduction property of wavelets in these areas was reported by the authors. Nath et al. [26] tested the algorithms incorporating wavelet based compression and denoising for preprocessing optical tomography reconstructions data. Reasonably good performance of wavelet based techniques in these applications has been reported.

Kaufmann and coworkers [27,28] used scale space filter, wavelet thresholding, and multiplicative correlation along with weighted smoothing splines for denoising the Digital Speckle Pattern Interferometric (DSPI) fringes. A comparative study of wavelet thresholding methods for denoising electronic speckle pattern interferometric fringes has also been reported [29]. Recently, Shakher et al. have proposed wavelet filtering scheme for speckle noise reduction in DSPI [30]. The application of the scheme for various engineering applications such as for vibration analysis [31,32], measurement of static and dynamic deformations [33], and mode shape visualization [34] has been reported.

In the present communication, we investigate the applicability of the preprocessing schemes and wavelet filters in analyzing the Lau interferometric fringes. The processing schemes and the wavelet filtering scheme is then applied to the interferometric fringe data obtained through Lau based tilt measuring set-up. It has been observed that improved measurement accuracy and the precision is obtained using the above technique.

2. Theory

Wavelet transforms have been successfully applied to many areas including tomographic reconstruction, image compression, noise reduction, image enhancement, texture analysis/segmentation and multi-scale registration. Wavelet transform has emerged as a robust mathematical tool for signal and image processing applications such as filtering, compressing, coding and denoising [35]. Wavelet transform means a small wave of limited duration and it can be a real or complex function with a zero average and localized both in time, space and frequency. Wavelet transform is extensively used for the processing of non-stationary signals due to its property of being a multi resolution and it offers a good localization in the time-frequency domains [36–38]. A non-stationary signal is a signal whose frequency content change in time/space. Fringe pattern resemble non-stationary signals [17]. Wavelet transform has developed as an alternative tool to the standard transform such as Fourier transform, to analyze fringe pattern. It's multi resolution property in time, space and frequency domain, overcomes the resolution problem in other transform.

The Lau effect is an interference phenomenon observed at infinity when light from a spatially and temporally incoherent source passes through two diffraction gratings separated by a distance of $Z = md^2/\lambda$, where m, d, and λ represent an integer, the period of the grating, and average wavelength of light used, respectively. Recently, the Lau effect has been used for measurement of small tilt angle by us [8]. The accuracy of the technique is determined mainly by the accuracy involved in the measurement of the inclination angle of the recorded moiré fringes. Because of the presence of several parameters degrading the image quality and the noise present, the accurate determination of inclination angles has been difficult to make. The attempt has been made to incorporate wavelet filtering techniques in the interferogram analysis, so as to improve the accuracy and precision of measurement.

The following filtering schemes/procedures based on Haar, Daubechies and Symlet wavelets have been tested on Lau fringes. A brief account of these is presented herewith.

2.1. Haar wavelet

The Haar wavelet is the oldest and simplest possible wavelet. Haar wavelet is discontinuous and resembles a step function. It can be represented as

$$\Psi(t) = \begin{cases} 1 & 0 \le t < 1/2, \\ -1 & 1/2 \le t < 1, \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

The scaling function of Haar wavelet, $\phi(t)$ can be described as

$$\phi(t) = \begin{cases} 1 & 0 < t < 1, \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Haar functions are a compact and support orthonormal wavelet system that satisfy Daubechies conditions of orthogonality. The disadvantage of the Haar wavelet is that it is not continuous and therefore not differentiable.

2.2. Daubechies (dbN) wavelets

The Daubechies wavelets are the most commonly used set of discrete wavelet transforms formulated by the Belgian mathematician Ingrid Daubechies in 1988. This formulation is based on the use of recurrence relations to generate progressively finer discrete samplings of an implicit mother wavelet function such that each resolution is twice that of the previous scale. These represent a family of orthogonal wavelets defining a discrete wavelet transform and characterized by a maximal number of vanishing moments for some given support. An orthogonal wavelet is a wavelet where the associated wavelet transform is orthogonal. That is the inverse wavelet transform is the adjoint of the wavelet transform. With each wavelet type of this class, there is a scaling function (also called father wavelet) which generates an orthogonal multi resolution analysis. The first wavelet of this family is a Haar wavelet. There is no explicit expression for these wavelets, however if a polynomial function defined as

$$P(y) = \sum_{k=0}^{N-1} C_{k=0}^{N-1+k} y^k \tag{3}$$

where C_k^{N-1+k} denotes the binomial coefficients, is assumed then the transfer function or filter function as per wavelet theory may be defined as,

$$|m_0(\omega)|^2 = \left(\cos^2\left(\frac{\omega}{2}\right)\right)^N P\left(\sin^2\left(\frac{\omega}{2}\right)\right) \tag{4}$$

where

$$m_0(\omega) = \frac{1}{\sqrt{2}} \sum_{k=0}^{2N-1} h_k e^{-ik\omega} \tag{5}$$