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Calculation of point-spread function for optical systems with finite value of numerical aperture

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Abstract

The scalar theory is described for calculation of the point-spread function (PSF) of optical systems with a large numerical aperture. In our work, we introduced an analytic description of this phenomenon and we derived exact and simple approximate relations that enable to calculate the PSF for a diffraction-limited optical system with a finite value of numerical aperture. The derived relations convert to classical relations that are commonly described in literature for the case that the value of numerical aperture tends to zero. Our derived relation is a generalization of the classical relation for calculation of PSF in the case of a large numerical aperture.

Keywords: Optical imaging; Scalar diffraction theory; Point-spread function; Numerical aperture

1. Introduction

The point-spread function (PSF) is a basic characteristic of imaging properties of optical systems. It is closely related to the resolving power and the optical transfer function of an optical system. In optical literature [1–3], the PSF is given only for the case of optical systems with a very small value of numerical aperture. These relations are accurate enough for a wide range of optical systems that one can meet in practice, because the value of numerical aperture for many optical systems (telescopes, camera lenses, etc.) is relatively small. For example, the camera lens with the *f*-number $c = 1.4$ has numerical aperture $\mathrm{NA} = 1/2c = 0.36$. However, the classical relation for the PSF calculation is not accurate enough for optical systems with a large value of numerical aperture. A typical representative of optical systems with the large numerical aperture is a microscope objective.

Diffraction in optical systems with a large numerical aperture is described in Refs. [4–14]. Refs. [15–21] deal with the application of Zernike polynomials for solving diffraction problems in the case of optical systems with aberrations.

The aim of this work is to show the effect of the numerical aperture value on the shape of the PSF of an optical system using the scalar wave theory. Furthermore, it is also derived a simple analytical expression for calculation of the PSF for the case of diffraction limited (aberration free) optical system with the circular exit pupil and with the numerical aperture of finite value.

2. Diffraction integral

Let us consider a scalar wave field. The applicability of the scalar analysis is limited to optical systems with a

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numerical aperture value of the order of NA<0.7. Beyond these values polarization effects become significant [6–11]. As it is well-known from the theory of electromagnetic field [1–6], the amplitude of the wave field $U(P)$ in an arbitrary point P of area bounded by the surface S can be determined, if the amplitude of the wave field $U(M)$ on this surface is known . It can be calculated from the diffraction integral

$$U(P) = -\frac{\mathrm{i}}{\lambda}\iint_S U(M)\frac{\mathrm{e}^{\mathrm{i}kr}}{r}\cos(\mathbf{n},\mathbf{r})\,\mathrm{d}S, \quad (1)$$

where M is the point lying on the surface S, $\mathbf{r}$ is the vector given by points P and M, $r = \|\mathbf{r}\|$ is the distance between points P and M, $\cos(\mathbf{n},\mathbf{r})$ is the cosine of the angle between the normal $\mathbf{n}$ to surface S and the position vector $\mathbf{r}$, $k = 2\pi/\lambda$ is the wavelength of light in given optical medium.

Let us now calculate the integral (1) for the case of an optical system with aberrations. The surface S is identical with the wave-front that exits the optical system and the point $M(x, y, z)$ is an arbitrary point lying on the wave-front S. The point $P(x_P, y_P, z_P)$, which lies in the image plane of the optical system, is the point in which we want to calculate the amplitude of the wave field, and the point $P_0(x_0, y_0, z_0)$, which also lies in the image plane of the optical system, is the center of the reference sphere with the radius R. It holds for the distance r of point P from point M.

$$r^2 = (x - x_P)^2 + (y - y_P)^2 + (z - z_P)^2. \quad (2)$$

Furthermore, the radius of curvature of the reference sphere can be expressed as:

$$R^2 = x_0^2 + y_0^2 + z_0^2 \quad (3)$$

and the equation of the wave front S is given by

$$(R + W/n)^2 = (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2, \quad (4)$$

where $W(x, y)$ is the wave aberration of the optical system and n is the refractive index of the optical medium in the image space. If we denote

$$u = x_P - x_0, \quad v = y_P - y_0, \quad (5)$$

then by using Eqs. (2)–(5), we obtain

$$r^2 = \left(R + W/n\right)^2 - 2\left[(x - x_0)u + (y - y_0)v\right] + u^2 + v^2. \quad (6)$$

If the wave aberration W is much smaller than the radius of curvature R (which is always satisfied in practice), then by using the approximate formula [24]:

$$\sqrt{1 + a} \approx 1 + a/2$$

and Eq. (6), we have

$$r \approx R + \frac{W}{n} - \frac{(x - x_0)u + (y - y_0)v}{R} + \frac{u^2 + v^2}{2R}. \quad (7)$$

If the surface S is described by the equation $z = z(x, y)$, then it holds for the element $\mathrm{d}S$ of this surface

$$\mathrm{d}S = \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\,\mathrm{d}x\,\mathrm{d}y = \sqrt{D}\,\mathrm{d}x\,\mathrm{d}y. \quad (8)$$

Using Eq. (4), we obtain

$$D = A/B, \quad (9)$$

where

$$A = (R + W/n)^2 + \delta x^2 + \delta y^2 + 2[\delta x(x - x_0) + \delta y(y - y_0)],$$

$$B = (R + W/n)^2 - (x - x_0)^2 - (y - y_0)^2$$

and

$$\delta x = -\frac{R}{n}\frac{\partial W}{\partial x}, \quad \delta y = -\frac{R}{n}\frac{\partial W}{\partial y}$$

are transverse ray aberrations of the optical system. Previous relations can be simplified for small values of aberrations as follows:

$$D = A_1/B_1,$$

where

$$A_1 = 1 + 2\left[\delta x(x - x_0) + \delta y(y - y_0)\right]/R^2,$$

$$B_1 = 1 - \left[(x - x_0)^2 - \left(y - y_0\right)^2\right]/R^2.$$

If we denote

$$p = (x - x_0)/R, \quad q = (y - y_0)/R,$$

we obtain

$$\sqrt{D} = \frac{1 + (\delta x p + \delta y q)/R}{\sqrt{1 - p^2 - q^2}}. \quad (10)$$

Moreover, if we denote

$$F(p, q) = U(p, q)\sqrt{D}\,\exp(\mathrm{i}k_0 W), \quad (11)$$

$$s = n\frac{u}{\lambda_0}, \quad t = n\frac{v}{\lambda_0},$$

where $k_0 = 2\pi/\lambda_0$, and λ_0 is the wavelength of light in vacuum, Eq. (11) can be written as follows ($\cos(\mathbf{n},\mathbf{r}) \approx 1$):

$$U(s, t) = C\iint_S F(p, q)\,\mathrm{e}^{-2\pi\mathrm{i}(ps + qt)}\,\mathrm{d}p\,\mathrm{d}q, \quad (12)$$

where C is a constant. The relation (12) makes possible to determine the amplitude of the wave field in the image plane of optical system with the finite value of numerical aperture. It is clear from this relation that the field $U(s, t)$ is proportional to the Fourier transform of the function $F(p, q)$.