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Propagation properties of beams generated by Gaussian mirror resonator in uniaxial crystals

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Abstract

Based on the paraxial vectorial theory of beams propagating in uniaxially anisotropic media, we have derived the analytical propagation equations of beams generated by Gaussian mirror resonator (GMR) in uniaxial crystals, and given the typical numerical example to illustrate our analytical results. Due to the anisotropy crystals, the ordinary and extraordinary beams originated by incident beams generated by GMR propagate with different diffraction lengths, thus the linear polarization state and axial symmetry of the incident beams generated by GMR do not remain during propagating in crystals.

Keywords: Gaussian mirror resonator; Uniaxial crystal; Propagation

1. Introduction

Unstable resonator with variable reflectance mirror has been proposed [1,2] and successfully implemented in many gas and solid-state lasers [3,4] for many years. Mirror with a Gaussian radial variation of reflectivity presents many useful characteristics. For example, optical resonator with a Gaussian mirror offers advantages over standard unstable resonator of good mode discrimination, smooth output beam profile, and large mode volume [5]. Moreover, it provides the possibility of generating a class of Gaussian beam and flat-topped Gaussian beam. The beams generated by Gaussian mirror resonator (GMR) can be decomposed into a linear combination of the lowest-order Gaussian beam. Its propagation properties through a paraxial ABCD system have been discussed based on the scalar diffraction theory [6–8].

The propagation of laser beams in anisotropic media can be treated in solving Maxwell's equations in anisotropic nonmagnetic media. [9–13] The basic idea is that optical field in uniaxial crystals is regarded as the superposition of ordinary and extraordinary beams independently propagating through homogeneous media with refractive indexes n_o and n_e^2/n_o (n_o, n_e—ordinary and extraordinary refractive indices), respectively. Thus, through solving the boundary value problems of Maxwell's equations, the beam propagation equations in uniaxial crystals have been derived, and the methods have been applied to the propagation of Gaussian beams, Hermite- and Laguerre–Gauss beams and

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Bessel–Gauss beams in uniaxial crystals [9,10]. In this paper, we will mainly study the propagation of the beams generated by GMR in uniaxially anisotropic crystals. Based on paraxial propagation equations in uniaxial crystals, the closed-form propagation expression for the beams generated by GRM in uniaxial crystals are derived, and numerical calculation results and analysis are given.

2. Theoretical analysis

Geometry of the beams generated by GMR incidence and propagation in rutile crystal are shown in Fig. 1. And we assume that in a rectangular coordinate system, a three-dimensional beam generated by GMR, which linearly polarized in the x-direction, is incident on a uniaxial crystal at the plane $z = 0$. The optical axis of the crystal coincides with the z-axis, and the dielectric tensor of the crystal can be written as

$$\varepsilon = \begin{pmatrix} n_o^2 & 0 & 0 \\ 0 & n_o^2 & 0 \\ 0 & 0 & n_e^2 \end{pmatrix}, \tag{1}$$

where n_o and n_e are the ordinary and the extraordinary refractive indexes, respectively. The field of the beams generated by GMR at the entrance plane $z = 0$ is [7]

$$E(r,0) = \sum_{m=0}^{\infty} A_m \exp\left[-(2m\beta^2 + 1)\frac{r^2}{\omega_0^2}\right], \tag{2}$$

here $A_m = \alpha_m A$, $\alpha_0 = 1$, $\alpha_1 = -K/2$, and

$$\alpha_m = \frac{(2m-3)(2m-5)\ldots(3)(1)}{m!}\left(\frac{K}{2}\right)^m \quad (m \geqslant 2), \tag{3}$$

where A represents the amplitude of a conventional Gaussian beam, ω_0 is the beam waist, and β is a parameter which introduced for a simpler description is given by

$$\beta = \frac{\omega_0}{\omega_c}, \tag{4}$$

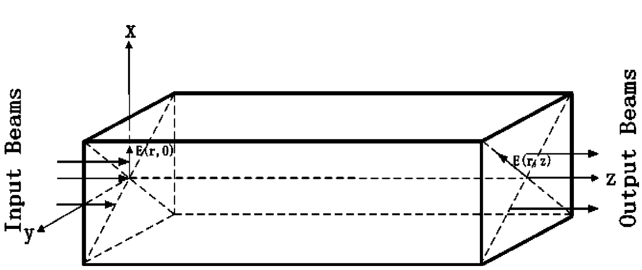

Fig. 1. Geometry of the beams generated by GMR incidence and propagation in rutile crystal.

and K is the on-axis (or peak) reflectance of the Gaussian mirror and ω_c is the mirror spot size at which the reflectance is reduced to $1/e^2$ of the peak value. We assume that the boundary electric field at the plane $z = 0$ is polarized along the x–axis:

$$\mathbf{E}(r,\phi,0) = E(r,0)\hat{\mathbf{e}}_x. \tag{5}$$

The electric field propagated for a distance z through the crystal is given by [12]

$$\begin{aligned} E_x(r,\phi,z) = {} & \exp(ik_0 n_0 z)\{A_o^{(0)}(r,z) + A_e^{(0)}(r,z) \\ & + \cos 2\phi[A_o^{(2)}(r,z) - A_e^{(2)}(r,z)]\}, \\ E_y(r,\phi,z) = {} & \exp(ik_0 n_0 z)\sin 2\phi[A_o^{(2)}(r,z) - A_e^{(2)}(r,z)], \end{aligned} \tag{6}$$

here $k_0 = \omega/c$ is the wave number in the vacuum. From Ref. [12], the field decomposition originated by the beams generated by GMR in uniaxial crystals obeys

$$A^{(0)}(r,z) = -i\frac{kn}{2z}\int_0^\infty \mathrm{d}r' r' E(r',0)\exp\left(-ikn\frac{r^2 + r'^2}{z}\right) \times J_0\left(\frac{knr'r}{z}\right), \tag{7}$$

$$A^{(2)}(r,z) = \frac{2}{r^2}\int_0^r d\xi\xi A^{(0)}(\xi,z) - A^{(0)}(r,z). \tag{8}$$

By setting $n = n_o$ and $n = n_e^2/n_o$ in these expressions, we can obtain the ordinary and the extraordinary beams inside the crystal. And by substituting Eq. (2) into Eq. (5), and using the integral formula: [14]

$$\int_0^\infty \exp(-pt)t^{q/2+n}J_q(2a^{1/2}t^{1/2})\mathrm{d}t = n!a^{q/2}p^{-(n+q+1)} \times \exp(-a/p)L_n^q(a/p), \tag{9}$$

we obtain

$$A_o^{(0)}(r,z) = -i\frac{v_o(z)}{4}\exp[-iv_o(z)r^2]\sum_{m=0}^{\infty} A_m Q_o(m,z)^{-1} \times \exp\left[-\frac{v_o^2(z)}{4Q_o(m,z)}r^2\right], \tag{10}$$

$$A_e^{(0)}(r,z) = -i\frac{v_e(z)}{4}\exp[-iv_e(z)r^2]\sum_{m=0}^{\infty} A_m Q_e(m,z)^{-1} \times \exp\left[-\frac{v_e^2(z)}{4Q_e(m,z)}r^2\right], \tag{11}$$