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Noise reduction for low-dose X-ray computed tomography with fuzzy filter

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ABSTRACT

Low-dose CT imaging has been particularly used in modern medical practice for its advantage on reducing the radiation dose to patients. However, reconstructed images will distinctly degenerate along with the decrease of the radiation dose. A resolution is to deal with the noisy projection space by an effective filter. This study was performed to address this problem and a fuzzy-median filter was proposed in this paper according to the properties of the noise of low-dose CT images. Reconstructed images by FBP were acquired from the previous noisy sinogram filtered by different filters for comparison. This fuzzy-median filter in fact is a spatially variant one that can solve the streak artifacts. And simulations also indicated that this spatially variant filter could suppress noise and obviously decrease streak artifacts in reconstructed images.

1. Introduction

As is known to all, X-ray CT (computed tomography) has been widely applied to many fields, such as medicine and industry. CT imaging is so useful in showing several types of tissue such as brain, bone and soft tissue that more and more scholars have applied themselves to this study. In CT imaging, X-ray radiation dose is vital and influences the quality of reconstructed images, namely high-dose X-ray acquires high-quality images, to the contrary, low-dose X-ray acquires low-quality images in which more streak artifacts appear. Thus, to increase radiation dose is an effective means to gain high-quality images. However, high-dose does harm to patients and leads to adverse health effects, and this is the reason why there is growing awareness of the significance of minimizing the radiation dose delivered to patients during X-ray CT [1]. And how to reduce noise has recently become a hotspot. Along with the appearance of anisotropic diffusion filter which was first described by Perona and Malik [2], many scholars started to improve this method for noise reduction and edge preservation in medical imaging. Demirkaya [3] used this nonlinear filter to suppress noise and preserve edge in CT images and Gerig et al. [4] applied this nonlinear filter to improve magnetic resonance. However, Saha and Udupa [5] pointed that there was an important disadvantage in the approach, that is, it does not provide image-dependent guidance for selecting an optimum gradient magnitude, and proposed a scale-based diffusive filter according to local scale information. Recently, Jing Wang et al. [6] proposed a model for the data mean and variance relation in sinogram space accounting for the noise properties of CT sinogram, and invested an alternative approach, instead of original methods, which treats the low-dose CT data noise in sinogram space.

In this paper, we investigated the anisotropic diffusion filter and provided a fuzzy-median filter for the sinogram space. Computer simulations were performed and showed that the proposed fuzzy-median filter had better reconstructed images with less streak artifacts.

2. Materials and methods

2.1. Noise model

Wang in [7] expounded that the noise modeling of the projection data. The projection data after system calibration and logarithm transformation are approximately Gaussian distributed in low-dose CT applications. And the proposed model described the relationship between the mean and the variance of the calibrated projection data as follows.

$$\sigma_{p_i} = f_i \exp\left(\frac{\bar{p}_i}{\eta}\right) \quad (1)$$

where $\bar{p}_i$ denotes the expectation value of the projection in detector bin i and σ_{p_i} denotes the corresponding variance. Notation η is a scaling parameter which is object-independent but completely determined by the system settings. And notation f_i is a parameter adaptive to different bins. It is obviously that the relationship between the data mean and the data variance is nonlinear.

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2.2. Anisotropic diffusion filter

Anisotropic diffusing derives from the classical thermal diffusion equation. Perona and Malik described the anisotropic diffusion filter for 2D images as follows:

$$\frac{\partial f(x,y,t)}{\partial t} = div(g(\left\|\nabla f(x,y,t)\right\|)\nabla f(x,y,t)) \tag{2}$$

where div represents the divergence operator, $\nabla f(x, y, t)$ represents the gradient operator, $f(x, y, t)$ stands for the image intensity at time t and $g(\left\|\nabla f(x,y,z)\right\|)$ is the diffusion coefficient function which controls the diffusion strength. Selecting the four-neighbor, the discrete formula can be expressed as:

$$f_{x,y}^{t+1} = f_{x,y}^{t} + \lambda \sum_{n=1}^{4} g(\nabla f_n^t)\nabla f_n^t \tag{3}$$

where λ is a scalar controlling the diffusion speed and n is the notation which indicating the four neighbors of pixel (x, y).

The diffusion coefficient function is a monotonically decreasing function of the gradient magnitude. It equals to zero when the gradient magnitude of the image intensity approaches to infinity and equals to one when the gradient magnitude is zero. Perona and Malik provided a kind of diffusion function that is expressed as:

$$g(s) = \frac{1}{1+(s^2/k^2)} \tag{4}$$

Fig. 1 describes the shape of Eq. (4) and demonstrates that the threshold k plays an important role in the diffusion process. The easiest method to estimate the threshold k is to empirically set it to a certain value. Torkamani-Azar and Tait [8] set the mean of the absolute gradient as k. Rousseeuw and Leroy [9] proposed gradient median method to estimate it from the view of robust statistics.

2.3. The fuzzy–median filter

Fuzzy theory has been applied to various aspects of image processing and the fuzzy membership is a vital factor in fuzzy theory. Zadeh lists fourteen kinds of membership in his literature≪Fuzzy Sets≫, and in this paper, we list some common membership functions which were often used in practice [10].

$$g(x) = \begin{cases} x\left(\dfrac{\sigma - x}{\sigma}\right)^2 & 0 < x < \sigma \\ 0 & |x| \ge \sigma \\ x\left(\dfrac{\sigma + x}{\sigma}\right)^2 & -\sigma < x \le 0 \end{cases} \tag{5}$$

$$g(x) = \begin{cases} 2\left(\dfrac{x}{\sigma}+1\right)^2 & -\sigma < x < \dfrac{-\sigma}{2} \\ 1-2\left(\dfrac{x}{\sigma}\right)^2 & |x| \le \dfrac{\sigma}{2} \\ 2\left(\dfrac{x}{\sigma}-1\right)^2 & \dfrac{\sigma}{2} < x \le \sigma \\ 0 & |x| \ge \sigma \end{cases} \tag{6}$$

$$g(x) = \begin{cases} 0.5+0.5\left(\dfrac{\pi}{\sigma - a}\left(x + \dfrac{\sigma - a}{2}\right)\right) & -\sigma < x < -a \\ 1 & |x| \le a \\ 0.5-0.5\left(\dfrac{\pi}{\sigma - a}\left(x - \dfrac{\sigma - a}{2}\right)\right) & a < x \le \sigma \\ 0 & |x| \ge \sigma \end{cases} \tag{7}$$

Fig. 2 depicts three membership functions and we got some important information comparing with Fig. 1 and Fig. 2. That's, the shape of the fuzzy membership is similar to that of the diffusion coefficient function. So, it is rational to consider diffusion coefficient function from the fuzzy theory and replace the diffusion coefficient function with the fuzzy membership. In the original anisotropic diffusion, it referred to the horizontal direction and vertical direction of neighborhoods. In this paper, we considered not only the horizontal and vertical directions but also the two diagonal directions, namely eight-neighborhood. In fact, this method was considered as a kind of generalized anisotropic diffusion. And we set membership values relative to the smooth region as the diffusion coefficient values of different directions. The membership used in this paper is described by the following formula:

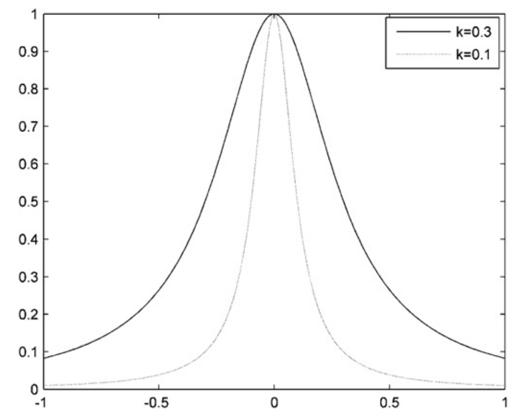

Fig. 1. Different shapes of diffusion coefficient function with different k.

$$g(\nabla f_{i,j}, d_{i,j}) = \exp\left(\frac{\nabla f_{i,j}^2}{\beta}\right) \quad \exp\left(\frac{-d_{i,j}^2}{2}\right) \tag{8}$$

where $\nabla f_{i,j}$ represents the deviation between the center pixel i and the neighboring pixel j, $d_{i,j}$ represents the Euclidean distance between neighboring pixel j and the central point i, β represents the scaling parameter and can be defined as the following formula:

$$\beta = \frac{1}{N-1}\sum_{j=1}^{N}(|\nabla f_{ij}|)^2 \tag{9}$$

notation N is the neighborhood of the pixel i in the projection space and is set to nine in this paper. A large β value indicates a non-smooth region and a small β value indicates a smooth region. Therefore, the generalized anisotropic diffusion model (also can be called a fuzzy filter) is described as follows:

$$f_i^{n+1} = f_i^n + \lambda g(\nabla f_{i,j}^n, d_{i,j})\nabla f_{i,j}^n \tag{10}$$

where n is the iteration number and λ is a scalar controlling the diffusion speed. With each iteration step, when $\nabla f_{i,j}$ is small, the resulting value of the membership function is distinctly large, namely, the degree that the center pixel i belongs to smooth region is large. And then the smoothing operation will be carried out and noise can be removed. To the contrary, when $\nabla f_{i,j}$ is large, the resulting value of the membership function is distinctly small, namely, the degree that the center pixel i belongs to smooth region is small and it may be at an edge. Then the diffusion process stops at the pixel i, and its gray-level is preserved. In addition, this membership function considers the distance filter which to some extent controls the degree of smoothing. Undoubtedly, this above method is very useful to remove noise at smooth region and to preserve edges. However, noise at edge region cannot be effectively removed because smoothing operation stops at edge.

Jian Ling [11] pointed out the fact that a noiseless image is usually insensitive to a median filter. This is because a median filter eliminates primarily sudden, transient spikes, while leaving sudden, sustained edges undisturbed. Moreover, the median filter does not significantly perturb the intensities in smooth regions.