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Multiple beam ring interference: A master curve for interference pattern design

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ABSTRACT

A Master Curve, capable of describing all possible configurations, was deduced by using the so-called Wigner Distribution Function, for a cylindrically-symmetric array of interfering beams, of the type utilized nowadays for optical tweezers and multiple-beam interference optical cages.

1. Introduction

There has been a recent interest on the so-called nondiffracting beams due to some of the potentially applicable characteristics, such as the preservation of the distribution of their transverse intensity along propagation, their self reconstruction after interacting with obstacle, the existence of phase dislocations in some cases [1–8] and self diffraction effects [9]. Nondiffracting beams have been recently found in periodic [10] and even liquid [11] media. Durnin [1] showed that the profile of nondiffracting beams corresponds, in general, to that of a zero-order Bessel profile. Today, cylindrical-incident Bessel beams have been studied experimentally and are used as optical tweezers [2–8]. In this work, the Wigner Transform for a multiple cylindrical array of beams is calculated, from which master interference plots are obtained. This master curves can be generalized and combined for designing specific patterns.

2. Multiple beam interference: *P*1 and *P*2 polarization

Let us consider a set of N beams $\vec{\Psi}_m$ impiging radially on a point onto a screen (Fig. 1), with the same degree of coherence, amplitude and phase. Each m wave is considered a plane wave. The equation for the interference intensity is then:

$$I_N\{x,y\} = \left|\sum_{m=1}^{N} \vec{\Psi}_m\{\Re^3, t\}\right|^2 = \sum_{m=1}^{N}\sum_{n=1}^{N} \vec{A}_m \cdot \vec{A}_n \exp[2\pi i(C_{m,n}x - Q_{m,n}y)] \tag{1}$$

Being ϕ the incidence angle of the beams, λ the wavelength and $\vec{A}_m$ the amplitude, and:

$$C_{m,n} = \frac{2}{\lambda}\cos\left[\frac{\pi(m+n)}{N}\right]\sin\left[\frac{\pi(m-n)}{N}\right]\sin[\phi] \tag{2}$$

$$Q_{m,n} = \frac{2}{\lambda}\sin\left[\frac{\pi(m+n)}{N}\right]\sin\left[\frac{\pi(m-n)}{N}\right]\sin[\phi] \tag{3}$$

We shall consider two cases of possible polarization: the first called $P1$ polarization (Fig. 2), when the polarization vector is parallel to the screen (radial polarization), and the second case, $P2$ polarization (Fig. 3), with the vector polarization perpendicular to the $P1$ polarization vector. The equations for the intensity of these two polarization are:

$$I_N\{P_1\} = A^2 \sum_{m=1}^{N}\sum_{n=1}^{N} \cos\left[\frac{2\pi(m-n)}{N}\right]\cos[2\pi(C_{m,n}x - Q_{m,n}y)] \tag{4}$$

$$I_N\{P_2\} = A^2 \sum_{m=1}^{N}\sum_{n=1}^{N} \left(\sin^2[\phi]\cos\left[\frac{2\pi(m-n)}{N}\right] + \cos^2[\phi]\right) \times\cos[2\pi(C_{m,n}x - Q_{m,n}y)] \tag{5}$$

We notice that the $P2$ polarization has an $P1$ component so we can rewrite this as:

$$I_N\{P_2\} = \sin^2[\phi] I_N\{P_1\} + A^2\cos^2[\phi] \sum_{m=1}^{N}\sum_{n=1}^{N} \cos[2\pi(C_{m,n}x - Q_{m,n}y)] \tag{6}$$

The first term on the right side of the equation can be regarded as the polarization vector parallel to the screen and thus the

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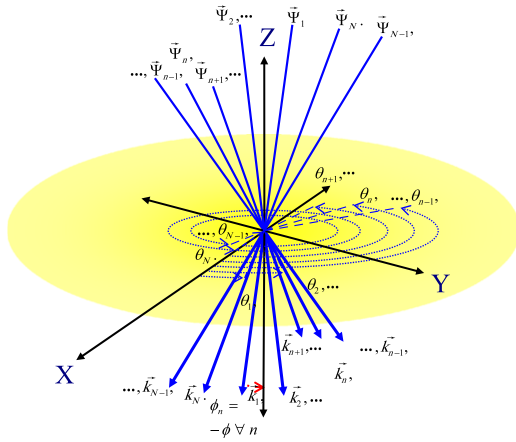

Fig. 1. N lightbeams coincide radially on a point, with cylindrical symmetry. A few lightbeams of the ensemble are shown.

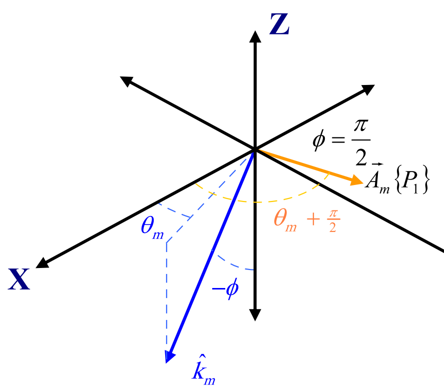

Fig. 2. P_1 Polarization. The polarization vector lies always in a plane parallel to the surface. $\vec{A}_m\{P_1\} = -A\sin[2\pi m/N]\hat{i} + A\cos[2\pi m/N]\hat{j} = A\hat{\theta}_m$.

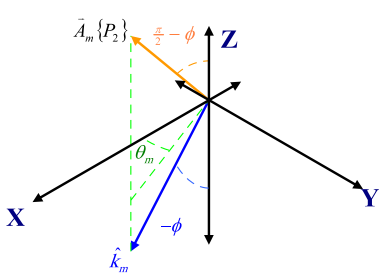

Fig. 3. P_2 Polarization. This polarization has a horizontal and a vertical component. $\vec{A}_m\{P_2\} = A(\cos[2\pi m/N]\sin[\phi]\hat{i} + \sin[2\pi m/N]\sin[\phi]\hat{j} + \cos[\phi]\hat{k}) = A\hat{\phi}_m$.

second one will be the perpendicular component, so we may rewrite these components as:

$$I_N\{P_1\} = I_N\{||\} \tag{7}$$

$$I_N\{\perp\} = A^2 \sum_{m=1}^{N}\sum_{n=1}^{N} \cos[2\pi(C_{m,n}x - Q_{m,n}y)] \tag{8}$$

and:

$$I_N\{P_2\} = \sin^2[\phi] I_N\{||\} + \cos^2[\phi] I_N\{\perp\} \tag{9}$$

Notice that Eq. (8) does not describes a physical case, but explains Eq. (9) in terms of horizontal and vertical polarization components. The plane joining the origin of the ensemble to each of the beams, and perpendicular to the incidence plane, can be described as an interference plane. There is always an interference pattern in the x (and thus u) direction, since one of the beams is always along this axis. We have assigned to the wavelength λ the value of one, so to get x in λ units. The Fourier Transforms (FT) of (7), (8) and (9) are:

$$\Im_{x,y}[I_N\{||\}] = \sum_{m=1}^{N}\sum_{n=1}^{N} \cos\left[\frac{2\pi(m-n)}{N}\right]\delta[u - C_{m,n}]\delta[v + Q_{m,n}] \tag{10}$$

$$\Im_{x,y}[I_N\{\perp\}] = \sum_{m=1}^{N}\sum_{n=1}^{N} \delta[u - C_{m,n}]\delta[v + Q_{m,n}] \tag{11}$$

$$\Im_{x,y}[I_N\{P_2\} = A^2\sin^2[\phi]\Im_{x,y}[I_N\{||\}] + A^2\cos^2[\phi]\Im_{x,y}[I_N\{\perp\}] \tag{12}$$

being u the frequency parallel to x and v the frequency parallel to y.

We calculate now the Wigner Transform (WT) of the ensemble of beams as:

$$W[\vec{\Psi}_N] = \sum_{m=1}^{N}\sum_{n=1}^{N} \vec{\Psi}_m(x+'x, y+y') \cdot \vec{\Psi}^*_n(x-x', y-y')\exp[-2\pi i(ux'+vy')]dx'dy' \tag{13a}$$

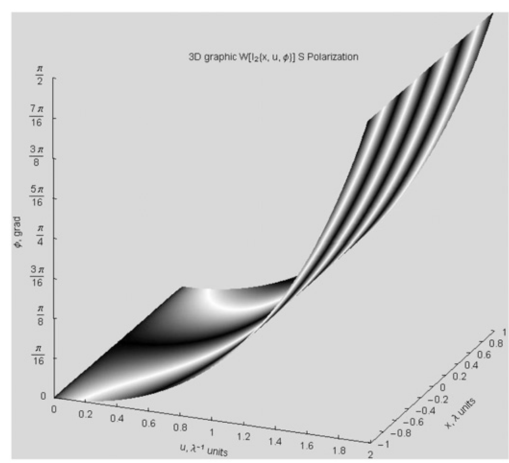

Fig. 4. 3-D graph showing the Wigner transform. It shows the relation between the x coordinate, its inverse frequency coordinate u and the angle of incidence of the beams φ for the intensity between two beams, $P1$ polarization. It is a surface where the intensity frequency lies on a horizontal line in the surface for each incidence angle.

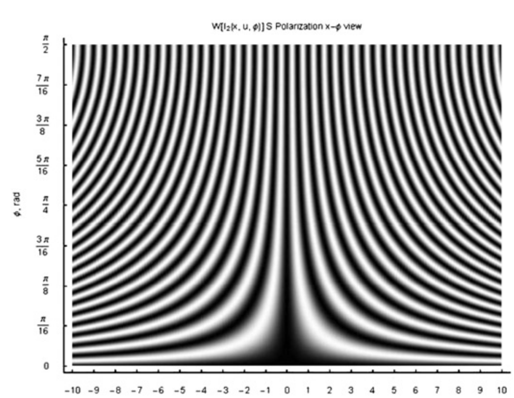

Fig. 5. 2-D view of the Wigner Transform of I2 $P1$ polarization in the $x-\varphi$ plane. It is a projection of the curved surface shown in Fig. 4. Since the zenithal angle is π, the angle between the two beams is 2φ.