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Transition from statistical Coulomb interactions to averaged space–charge effect

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ABSTRACT

The statistical Coulomb interaction effect and the averaged space–charge effect, which broaden electron-beam size, have been studied to clarify their applicable regions and their transition region as a function of beam current. The beam size of a single beam segment can be analytically estimated by two theories based on different approaches: the expression of the trajectory displacement effect based on stochastic electron–electron interactions, and the beam radius equation based on the Coulomb repulsion force by averaged space charge. Both theories were compared with Monte Carlo simulation results for a beam-current ranging from 0.1 nA to 10 mA. It was confirmed that the beam size at the end of the focusing segment could be predicted by the expression of the trajectory displacement effect in a low beam current and by the beam radius equation in a high beam current. In addition, the beam sizes predicted by these theories were found to result in almost the same for a medium beam current. The theories transited as the beam-current increased.

1. Introduction

Electron-beam apparatuses used for wafer inspection, device testing, and materials analysis require very fine electron-beam probes in order to achieve high spatial resolution. A probe with a large beam current is also required for inspection of high throughput and analysis of high signal-to-noise ratios. However, it is difficult to maintain a fine probe size in a high beam-current region because the Coulomb repulsion force becomes dominant with increasing beam current. Therefore, the Coulomb interaction should be properly evaluated for design of high-resolution and high-throughput probe-forming apparatuses.

The broadening of the probe beam size by the Coulomb repulsion can be analytically predicted by two theories based on different approaches. One theory considers the Coulomb force from an averaged space charge produced by a large number of electrons. A cylindrical electron beam is broadened by the averaged space charge. This effect can be described as a ray equation [1]. In Refs. [2,3], the beam radius equation including not only the Coulomb effect but also the effect of the initial electron velocity spread is shown. This equation including the averaged space–charge effect is assumed not to be valid when there are few electrons around a ray electron.

The other theory considers statistical Coulomb interactions among discrete electrons in a beam. A single electron traveling in a beam is affected by all the other electrons randomly distributed in that beam. Thus, the displacement of an electron ray is not deterministic but stochastic. Jansen deduced analytical expressions for the trajectory displacement effect and the Boersch effect under his extended two-particle model [4,5]. In this theory, most Coulomb interactions are required to be weak, and only a single strong collision is covered. Therefore, this theory is valid for the low and medium beam-current regions. There are many other statistical theories with different approaches, which are summarized in Ref. [4].

The theories based on the averaged space–charge effect and the statistical Coulomb interaction effect are expected to have different validity regions regarding the beam parameters. The aim of this paper is to clarify the applicable regions and the transition region of the theories for predicting probe-size broadening. For this purpose, the beam sizes estimated by both the theories are compared with the values calculated by a Monte Carlo simulation, depending especially on the beam current.

2. Review of analytical theories for beam-size broadening

Beam-size broadening by the averaged space–charge effect in a field-free drift space is described by a ray equation including an averaged charge-density distribution [1,4]. With assumptions of a laminar flow and uniform distribution of electrons in a beam, the beam radius R satisfies the equation for a cylindrical beam along the z-axis:

$$\frac{d^2R}{dz^2} = \frac{1}{4\pi\varepsilon_0}\sqrt{\frac{m}{2e}}\frac{I}{V^{3/2}}\frac{1}{R}, \quad (1)$$

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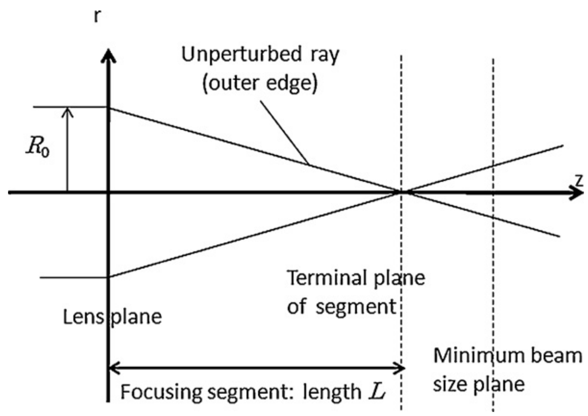

Fig. 1. Focusing segment for beam-size estimation. Beam size produced by unperturbed beam at terminal plane of segment becomes zero. Plane for obtaining minimum beam size of perturbed beam slightly shifts around terminal plane.

where m, e, ε_0, I, and V are the electron mass, electron charge, vacuum permittivity, beam current, and beam voltage, respectively. The relativistic effect is ignored. If the initial velocity spread of a beam is considered, the beam radius equation is generalized by taking the beam emittance into account [2,3]. If an electrostatic field is present and a magnetic field is absent in a beam segment, the beam radius equation is described as

$$\frac{d^2R}{dz^2}+\frac{1}{2V}\frac{dV}{dz}\frac{dR}{dz}+\frac{1}{4V}\frac{d^2V}{dz^2}R-\frac{1}{4\pi\varepsilon_0}\sqrt{\frac{m}{2e}}\frac{I}{V^{3/2}}\frac{1}{R}-\frac{2}{meV}\left(\frac{\varepsilon^*}{\pi}\right)^2\frac{1}{R^3}=0. \tag{2}$$

ε^* is normalized beam emittance, which may be set to

$$\varepsilon^*=\pi\sqrt{2mkT}\cdot R_0, \tag{3}$$

where k, T, and R_0 are the Boltzmann constant, temperature, and radius of an emission area, respectively. The laminar flow condition is lost.

The focusing segment shown in Fig. 1 was considered. An unperturbed beam with a radius R_0 is focused by a thin lens at the terminal plane of the segment with a focal length of L. The beam radius at the terminal plane can be obtained by numerically integrating Eq. (2). In cases of no external field ($dV/dz=0$) and no initial velocity spread ($\varepsilon^*=0$), the beam radius R_t at the terminal plane is approximated as

$$R_t\sim\frac{1}{8\pi\varepsilon_0}\sqrt{\frac{m}{2e}}\frac{IL^2}{R_0V^{3/2}} \tag{4}$$

under the condition of

$$\frac{I}{V^{3/2}}\gg 2\pi\varepsilon_0\sqrt{\frac{2e}{m}}\frac{R_0^2}{L^2}. \tag{5}$$

The minimum beam radius R_m is obtained at a different plane because of space–charge defocusing. R_m is exactly evaluated as

$$R_m=R_0\exp\left(-\frac{1}{2\pi\varepsilon_0}\sqrt{\frac{m}{2e}}\frac{R_0^2}{L^2}\right). \tag{6}$$

The beam size of full width median value (FW_{50}) becomes

$$FW_{50}=0.80795\times R \tag{7}$$

under a condition of uniform electron distribution.

Beam-size broadening by statistical Coulomb interactions is given as the expression of the trajectory displacement effect by Jansen [5]. In this theory, stochastic displacement of a test electron running along the center axis of a beam by the Coulomb repulsions from randomly distributed field electrons is statistically calculated in a field-free segment. Although the average of each displacement of the test electron should be zero due to the axially symmetric distribution of the field electrons, finite deviation of the displacement occurs.

The scaled trajectory displacement (FW^*_{50}) at an arbitrary plane indicated by a location parameter S_i is given in Eq. (17) of Ref. [5] as

$$FW^*_{50}=4.2917H_{CT}(\bar{\lambda},v^*_0,r^*_c,S_c,S_i)\bar{\lambda}^{2/3}. \tag{8}$$

Here, $\bar{\lambda}$, v^*_0, r^*_c, S_c, and S_i are scaled linear electron density, scaled transverse velocity, scaled crossover radius, crossover location parameter, and image plane location parameter, respectively. The scaled parameters are defined as

$$FW^*_{50}=FW_{50}\cdot\left(\frac{2\pi\varepsilon_0}{e}\right)^{1/3}\frac{V^{1/3}}{L^{2/3}}, \tag{9}$$

$$\bar{\lambda}=\frac{m^{1/2}}{2^{7/2}\pi\varepsilon_0e^{1/2}}\frac{I\cdot L^2}{R_0^2V^{3/2}}, \tag{10}$$

$$v^*_0=R_0\cdot\left(\frac{2\pi\varepsilon_0}{e}\right)^{1/3}\frac{V^{1/3}}{L^{2/3}}, \tag{11}$$

$$r^*_c=r_c\cdot\left(\frac{2\pi\varepsilon_0}{e}\right)^{1/3}\frac{V^{1/3}}{L^{2/3}}. \tag{12}$$

The function H_{CT} is also defined in Ref. [5].[1] The beam size is obtained by adding the unscaled trajectory displacement (FW_{50}) to the beam size of an unperturbed trajectory. The beam size at the terminal plane shown in Fig. 1 is directly estimated from Eq. (32) in Ref. [5] as

$$FW_{50}=2.2135\times10^{-20}H_{CT}(\bar{\lambda},v^*_0,r^*_c,S_c,S_i)\frac{I^{2/3}L^2}{V^{4/3}{R_0}^{4/3}} \tag{13}$$

using $r_c<10^{-12}$, $S_c=0.999$, and $S_i=1$ because the beam size produced by the unperturbed trajectory is zero. A minimum beam size is also obtained at the terminal plane as there is no space–charge defocusing for a test electron. If a uniformly external electrostatic field is present, Eq. (13) should be modified. The expression deduced by a slice method [4] is described in Appendix A.

The Boersch effect, which causes the energy broadening produced in a beam segment, can also broaden the beam size through a chromatic aberration of a lens. However, in the single segment shown in Fig. 1, the Boersch effect does not contribute to the beam-size broadening because no lens action occurs during transmitting in the segment. Beam-size broadening by the Boersch effect should be considered in the case of successive segments combined by a lens.

3. Comparison of theories by Monte Carlo method

3.1. Monte Carlo method

The beam sizes predicted by the averaged space–charge effect (the beam radius equation) and by the statistical Coulomb interaction effect (the expression for the trajectory displacement effect) were compared with the beam size calculated by a Monte Carlo simulation method in order to obtain the applicable regions of the theories and the transition behavior. The Monte Carlo method is a computer simulation method for tracing rays of randomly distributed electrons with a short time step of Δt. Since the Coulomb interactions among electrons running in a vacuum are determined

[1] The constant C included in Eq. (34) in Ref. [5] should be replaced with $1/C$.