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The probability of orbital angular momentum states of single photons with Z-tilt corrected residual aberration in a slant path turbulent atmosphere

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ABSTRACT

With Rytov approximation and Kolmogorov spectrum model of index-of-refraction fluctuation, we study the effects of turbulence aberration on the orbital angular momentum of single photons in atmospheric communication channel. A theoretical model of measurement probabilities of orbital angular momentum states for single photons propagation under the Zernike tilt corrected slant path turbulent atmosphere channel is established. Our research shows that tilt-corrected residual aberration not only damage the initial OAM, but also induce new OAM. With the increasing of D/ρ_0, the number of the initial OAM photons will go down while the effective number of new OAM ascends. Meanwhile, the receiving probability of initial OAM photons declines, as the turbulence shifts from weak to strong. For Zernike tilt-corrected residual aberration, the receiving probability of initial OAM photons declines as the diameter of detector increases. The effect of Zernike tilt-corrected residual aberration on OAM of the photons is more larger than the effect of Zernike tilt turbulent aberration.

1. Introduction

Several proposals have recently been made to use the orbital angular momentum (OAM) states of light as a basis set for impressing quantum information onto single-photon light field [1,2]. A key motivation for this idea is that the OAM states provide an infinite orthonormal basis set for describing the transverse structure of the beam [3]. This property may be exploited with the context of optical communications. The orthogonality among beams with different OAM states allows the simultaneous transmission of information from different users, each on a separate OAM channel. For atmosphere optical communication, however, it is important to note the perturbation of atmospheric turbulence on OAM states [3–7]. Many investigation results for the effects of atmospheric turbulence on the OAM states of photons propagation in atmosphere optical communication channel have been reported, such as, Paterson [5,6] investigated the effect of Kolmogorov atmospheric turbulence aberrations on free-space optical communication using angular momentum states of single photons with the pure phase perturbation approximation of turbulent aberrations. Anguita et al. [7] numerically analyzed the effects of beams and find that turbulence induces attenuation and crosstalk among channels. Glenn et al. [8] analyzed the influence of atmospheric turbulence on propagation of quantum states of light carrying orbital angular momentum and given a result that quantifies the rate at which quantum information encoded the OAM states of individual photons is lost as a result of propagation through atmospheric turbulence. Aksenov [9] studied the effect of Kolmogorov turbulence on the orbital angular momentum of the Laguerre–Gaussian beam propagating through the atmosphere with classical electromagnetic theory. As our known, there are no reports about the probability model of orbital angular momentum states of photons with Z-tilt corrected turbulence phase aberration, but in free space quantum atmospheric communication system, the tilt or jitter of beam are compensated or tracked by adaptive optics system or tracking and pointing (ATP) system [10–12].

In the current study, we analyze the effects of residual turbulence-aberration of Zernike corrected system on optical communication with OAM states based on the Zernike polynomial expansion of turbulence-induced phase aberrations. In this study, the Kolmogorov turbulence spectrum of index-of-refraction fluctuation and Rytov approximation are considered. The probability model of orbital angular momentum states of photons with Zernike tilt corrected turbulence phase aberration is presented.

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2. Probability of orbital angular momentum states

The free-space Gauss–Laguerre (LG) beam modes $LG_{l_0,p}(\rho,\varphi,z)$ have transverse wavefunction which are eigenfunctions of orbital angular momentum operator $\hat{L}_z = -i\hbar\partial/\partial\varphi$ [1,2], therefore, we make use of the LG beam modes to study the effects of atmospheric turbulence on the orbital angular momentum of the transmitted photon in quantum communication channel. Using cylindrical polar coordinates (ρ, φ, z) and defining the z-axis as the propagation direction. The normalized $LG_{l_0,p}$ model of collimated beam at z is given by [1]

$$LG_{l_0,p}(\rho,\varphi,z) = R_{l_0,p}(\rho,z)\frac{\exp(il_0\varphi)}{\sqrt{2\pi}}\exp\left[-i\left(2p+\left|l_0\right|+1\right)\delta\right] \quad (1)$$

where the parameters l_0 and p are the radial and azimuthal mode indices, respectively. l_0 is also the orbital angular momentum quantum number for photons in the mode. φ is the azimuthal angle, ρ is the radial cylindrical coordinate and δ is the Gouy phase. The radial orthonormal basis functions $R_{l_0,p}(r)$ of the field distribution of the LG beam model with eigenvalues of orbital angular momentum $l_z = l_0\hbar$ are of the form

$$R_{l_0,p}(\rho,z) = \frac{1}{w}\sqrt{\frac{p!}{(p+\left|l_0\right|)!}}\left(\frac{\rho}{w}\right)^{|l_0|}L_p^{l_0}\left(\frac{\rho^2}{w^2}\right)\exp\left(-\frac{\rho^2}{2w^2}\right)\exp\left(-\frac{ik\rho^2}{4R}\right) \quad (2)$$

where $L_p^{l_0}(\cdot)$ are the generalized Laguerre polynomials. $w = w_0\sqrt{1+\left(z/z_R\right)^2}$ is the spot size of bean, $z_R = \frac{1}{2}kw_0^2$ is Rayleigh distance, w_0 is beam waist and $R = z\left[1+\left(z_R/z\right)^2\right]$ is phase front radius of curvature.

For Zernike corrected optical communication channel, the Rytov approximation [13] can be used. The complex amplitude of Laguerre–Gauss beam model which is propagation in the weak turbulence regime and at z is then given by

$$LG_{l,p}(\rho,\varphi,z) = LG_{l_0,p}(\rho,\varphi,z)\exp\left[\psi_{co}(\rho,\varphi,z)\right] \quad (3)$$

where $\psi_{co}(\rho,\varphi,z) = \chi(\rho,\varphi,z) + i\left[S(\rho,\varphi,z) - S_{ti}(\rho,\varphi,z)\right]$ is the turbulent Zernike tilt corrected complex phase, $S_{ti}(r,\varphi) = 2a_2\rho\cos\varphi + 2a_3\rho\sin\varphi$ is the Zernike tilt wavefront aberration function [14], $a_{[2,3]}$ are the 2nd or 3rd coefficients of Zernike polynomial expansion, $\left\langle a_{[2,3]}^2\right\rangle = 0.048\left(D/r_0\right)^{5/3}$, D is sampling diameter of wavefront, r_0 is the Fried's coherence length.

As the beam propagates through the atmosphere, the beam is scattered by refractive index inhomogeneous which may alter its angular momentum, that is, if a photon containing $l_0\hbar$ units OAM were transmitted, as a result of atmospheric turbulence the received photon is measured to carry OAM of $l\hbar$ with $l_0 \neq l$. The effect of the refractive index fluctuations perturbs the complex amplitude of the wave so that it is no longer guaranteed to be in the original eigenstate of orbital angular momentum. The resulting wave is now written as a superposition of eigenstates.

$$LG(\rho,\varphi,z) = \sum_p\sum_l a_{l,p}(z)LG_{l,p}(\rho,\varphi,z) \quad (4)$$

where $a_{l,p}(z)$ is the expansion coefficient,

$$a_{l,p}(z) = \iint R_{l,p}^*(\rho)\frac{\exp(-il\varphi)}{\sqrt{2\pi}}\exp\left[i\left(2p+\left|l\right|+1\right)\delta\right] LG(\rho,\varphi,z)\rho d\rho d\varphi \quad (5)$$

here * denotes complex conjugate.

The probability of obtaining a measurement of the orbital angular momentum $l_z = l\hbar$ is obtained by summing the probability associated with that eigenvalue and taking the ensemble average over the turbulent aberrations [5,6]

$$P(l) = \left\langle\sum_p\left|a_{l,p}(z)\right|^2\right\rangle_{atm} = \int\iint\left\langle LG^*(\rho,\varphi',z)LG(\rho,\varphi,z)\right\rangle_{atm}\rho d\rho\frac{\exp\left[il(\varphi-\varphi')\right]}{\sqrt{2\pi}}d\varphi' d\varphi \quad (6)$$

where $\langle\cdot\rangle_{atm}$ denotes the ensemble average.

As in Refs. [5,6], it is assumed that the statistics of atmosphere-turbulence aberration is isotropic and since the beam profile at launch was rotationally symmetric, then the field correlation be written as

$$C_{LG}(\rho,\Delta\varphi,z) = \left\langle LG^*(\rho,0,z)LG(\rho,\Delta\varphi,z)\right\rangle \quad (7)$$

where $\Delta\varphi = \varphi - \varphi'$. In Eq. (6), making the substitution $\varphi = \Delta\varphi + \varphi'$, we have the orbital angular momentum measurement probability

$$P(l) = \iint\iint C_{LG}(\rho,\Delta\varphi,z)\rho d\rho\exp\left[il\Delta\varphi\right]d\Delta\varphi \quad (8)$$

Substituting Eq. (3) into Eq. (8) gives

$$C_{LG}(\rho,\Delta\varphi,z) = \left|R_{l_0,p}(\rho,z)\right|^2\frac{\exp(il_0\Delta\varphi)}{2\pi}\left\langle\exp\left\{-\frac{1}{2}D_{\psi_{co}}(\rho,\Delta\varphi)\right\}\right\rangle \quad (9)$$

where $D_{\psi_{co}}(\rho,\Delta\varphi) = D_{\psi_{co}}\left(\left|2\rho\sin\left(\Delta\varphi/2\right)\right|\right)$ is the wave-structure function. For Kolmogorov turbulence the tree-dimensional spectrum of index-of-refraction fluctuation is [13]

$$\phi_n(\kappa) = 0.033C_n^2(z)\exp\left(-\frac{\kappa^2}{\kappa_m^2}\right)\kappa^{-11/3}, \quad \kappa_m = \frac{5.92}{l_0} \quad (10)$$

where l_0 is inner scale of atmospheric turbulence, $C_n^2(z)$ is the structure constant of the fluctuation of the index of refraction, as a function of the path z. One of the most widely used models is the Hufnagel–Velly (H–V) model described by [13]

$$C_n^2(z\cos\theta) = 0.00594\left(v/27\right)^2\left(z\cos\theta\times 10^{-5}\right)^{10}\exp\left(-z\cos\theta/1000\right) + 2.7\times 10^{-16}\exp\left(-z\cos\theta/1500\right) + C_n^2(0)\exp\left(-z\cos\theta/100\right) \quad (11)$$

where $z\cos\theta = h$ is altitude, $v = 2.1$ m/s is the rms wind speed, $C_n^2(0)$ is the structure parameter at the ground. A commonly use value for the $C_n^2(0)$ parameter is $C_n^2(0) = 1.7\times 10^{-14}\ \mathrm{m}^{-2/3}$ and θ is the zenith angle of communication channel.

Probability of orbital angular momentum states (8) can be write as

$$P(l) = \frac{1}{2\pi}\int_0^\infty\left|R_{l_0,p}(\rho,z)\right|^2\left\{\int_0^{2\pi}\exp\left[-\frac{1}{2}D_{\psi_{co}}\left(\left|2\rho\sin\left(\Delta\varphi/2\right)\right|\right)\right]\times\exp\left[-i(l-l_0)\Delta\varphi\right]d\Delta\varphi\right\}\rho d\rho \quad (12)$$

2.1. Zernike tilt-corrected phase-structure function

The Zernike tilt-corrected complex phase at the point (ρ,φ,z) is given by [15]

$$\psi_{co}(\rho,\varphi,z) = \psi(\rho,\varphi,z) - iS_{ti}(\rho,\varphi,z) = \chi(\rho,\varphi,z) + i\left[S(\rho,\varphi,z) - S_{ti}(\rho,\varphi,z)\right] \quad (13)$$