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Focusing properties of Gaussian beam with mixed screw and conical phase fronts

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ABSTRACT

Vector diffraction theory is employed to investigate the focusing properties of the Gaussian beams with mixed screw and conical phase fronts. Numerical simulations show that the Gaussian beams with screw–conical phase fronts are different from both the ordinary Laguerre–Gaussian beams and the higher-order Bessel beams. Rather than forming the ring-shaped intensity distributions characteristic of optical vortices, focusing the Gaussian beams with screw–conical phase fronts produce non-symmetric spiral intensity distributions at the focal plane. The intensity distribution forms a counter-clockwise non-symmetric screw path around the focus. The rotation of intensity distributions was observed in the focal plane. The gradient force patterns of these beams focused with high NA are also investigated. The results show that the gradient force pattern shape depends principally on parameter topological charge n of the phase distribution. The gradient force pattern expands with increase in the parameter m of the phase distribution. Therefore, one can change the topological charge n or the parameter m of the phase mask to construct the tunable optical trap to meet different requirements. Its potential application might include rotational positioning of particles and accumulation of smaller non-symmetric particles towards the focus.

1. Introduction

Light beams with helical phase fronts can be described by an azimuthally phase term of $\exp(-il\phi)$. For integer l values, the phase fronts for a given phase comprise l intertwined helical surfaces giving a screw dislocation along the beam axis, and a resulting annular intensity cross-section. This screw-phase dislocation along the axis is an optical vortex of charge l. Light beams with screw-phase dislocation focus to rings rather than points, and also carry an orbital angular momentum [1,2]. Such beams are most often referred to as optical vortices. One particular application that takes advantage of this intensity distribution is the trapping of particles with either high or low refractive indices using strongly focused optical vortices [3]. There are several families of optical beams, such as Laguerre–Gaussian beams [4] and higher-order Bessel beams [5,6] that may be described as optical vortices. Recently, there has been significant interest in fractional half-charge dislocations embedded within optical beams [7–9]. To date, for the generation of an optical beam of half-integer fractional charge, spiral phase mask [8,9], off-axis holograms, and spatial light modulator [10] have been used, such as helico-conical beams [10], Helmholtz–Gaussian beams [11,12], hypergeometric beams [13], etc. However, the structural properties of individual beams are radically changed via combining them into on-axis [14] or off-axis [15–19] compositions that can lose their structural stability. In this article, we focus on analyzing the focusing properties of the Gaussian beam with screw–conical wavefront by vector diffraction theory. Screw–conical phase fronts may be generated by computer-generated holograms or by placing spatial light modulator in the laser path, which can conveniently alter the wavefront phase distribution of the incident laser beam in the control of computer. The paper is organized as follows: Section 2 gives the principle of the optical focusing system and Section 3 presents the discussions and results. The conclusions are summarized in Section 4.

2. Principle of the focusing system

In the focusing system, we investigate whether a circular phase mask can change the phase of an incident beam with screw–conical distribution in the transverse plane. The phase mask is placed in front of the aperture plane of the lens; then the modulated beam with screw–conical phase distribution is focused

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through an objective lens. Assume that the radius of the optical aperture is a, then the transmittance function of the phase mask can be written as

$$\rho(r,\ \varphi)=\begin{cases}\exp(-i\phi) & r<a\\ 0 & \text{otherwise}\end{cases} \tag{1}$$

where r and φ are the cylindrical coordinates, and phase variation in a circular phase mask with screw–conical distribution can be written as Eq. (2),which can be seen as superposition of screw and conical phases.

$$\phi(r,\varphi)=n\varphi\left(m-\frac{r}{a}\right) \tag{2}$$

The Gaussian beam with screw–conical wavefront is focused, and the intensity distribution in geometrical focal plane is the content of this article. The amplitude of the electric field before focusing lens is

$$E(r,\varphi)=A\exp\left(-\frac{r^2}{\omega^2}\right)\exp(-i\phi) \tag{3}$$

where A is a constant and ω is the waist width of the Gaussian beam. Through simple derivation, Eq. (3) may be rewritten as

$$E(r,\varphi)=A\exp\left(-\frac{(r/a)^2}{(\omega/a)^2}\right)\exp(-i\phi) \tag{4}$$

For $r=f\sin(\theta)$, here f is the focal length, so

$$E(r,\varphi)=E(\theta,\varphi)=A\exp\left(-\frac{[f\sin\theta/f\sin(\theta_{max})]^2}{(\omega/a)^2}\right)\exp(-i\phi) \tag{5}$$

$$E(\theta,\varphi)=A\exp\left(-\frac{\left[\dfrac{\sin\theta}{\sin(\theta_{max})}\right]^2}{(\omega/a)^2}\right)\exp(-i\phi) \tag{6}$$

Assume that the system investigated is in air. where $\theta\in[0, \arcsin(\text{NA})]$ is convergence angle. For $\text{NA}=\sin(\theta_{max})$, so

$$E(r,\varphi)=A\exp\left(-\frac{\sin^2\theta}{\text{NA}^2(\omega/a)^2}\right)\exp(-i\phi) \tag{7}$$

NA is the numerical aperture of the focusing system. It is assumed that the incident polarization is in x-axis direction. According to the energy conversation theory and vector diffraction theory, the amplitude of electric field in the focal region can be written as [20,21]

$$\vec{E}(\rho,\psi,z)=\frac{1}{\lambda}\iint_{\Omega}E(r,\varphi)\Big\{[\cos\theta+\sin^2\varphi(1-\cos\theta)x + \cos\varphi\sin\varphi(\cos\theta-1)y+\cos\varphi\sin\theta z\Big\} \times\exp[-ik\rho\sin\theta\cos(\varphi-\psi)]\exp(-ikz\cos\theta)\sin\theta d\theta d\varphi \tag{8}$$

where $\varphi\in[0, 2\pi]$. Vectors $\boldsymbol{x}$, $\boldsymbol{y}$ and $\boldsymbol{z}$ are the unit vectors in the x, y and z directions, respectively. It is clear that the incident beam is depolarized and has three components (E_i, E_j and E_k) in **x**, **y** and **z** directions, respectively. The variables ρ, ψ and z are the cylindrical coordinates of an observation point in the focal region. Substituting Eq. (7) into Eq. (8), we can get amplitude in the x direction,

$$E_i(\rho,\psi,z)=\frac{A}{\lambda}\int_0^{\theta_{max}}\int_0^{2\pi}\exp\left(-\frac{\sin^2\theta}{NA^2(\omega/a)^2}\right) \times\exp(-i\phi)[\cos\theta+\sin^2\varphi(1-\cos\theta)]_i \times\exp[-ik\rho\sin\theta\cos(\varphi-\psi)]\exp(-ikz\cos\theta)\sin\theta d\theta d\varphi \tag{9}$$

Amplitude in the y direction is

$$E_j(\rho,\psi,z)=\frac{A}{\lambda}\int_0^{\theta_{max}}\int_0^{2\pi}\exp\left(-\frac{\sin^2\theta}{NA^2(\omega/a)^2}\right) \times\exp(-i\phi)\cos\varphi\sin\varphi(\cos\theta-1) \times\exp[-ik\rho\sin\theta\cos(\varphi-\psi)]\exp(-ikz\cos\theta)\sin\theta d\theta d\varphi \tag{10}$$

Amplitude in the z direction is

$$E_k(\rho,\psi,z)=\frac{A}{\lambda}\int_0^{\theta_{max}}\int_0^{2\pi}\exp\left(-\frac{\sin^2\theta}{NA^2(\omega/a)^2}\right) \times\exp(-i\phi)\cos\varphi\sin\theta\exp[-ik\rho\sin\theta\cos(\varphi-\psi)] \times\exp(-ikz\cos\theta)\sin\theta d\theta d\varphi \tag{11}$$

3. Results and discussions

The optical intensity distribution in focal region is calculated as the squared modulus of amplitude distribution in focal region Eq. (8). Without losing generality and validity, the intensity is normalized by optical intensity maximum. In the figure, $V_x=k\rho\cos\theta$ and $V_y=k\rho\sin\theta$ and the unit of V_x and V_y coordinates is $\lambda/2\pi$, where$\lambda$is the wavelength of the incident beam. Fig. 1 illustrates the evolution of three-dimensional light intensity distribution for $m=1$, NA = 0.7 with changing parameter n. It can be seen that the three-dimensional distribution of light intensity changes considerably with parameter n. When parameter $n=2$, there are spiral intensity distributions and the maximum intensity peak stays at the spiral's head. The rotation of intensity distributions is evident in the focal plane. The intensity distribution forms a counter-clockwise non-symmetric screw path around the focus, as shown in Fig. 1(a). Increasing parameter n from 2 to 3, the intensity distributions are still

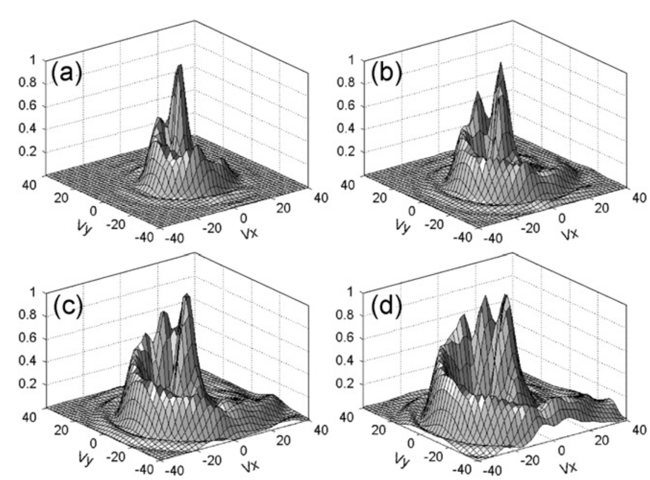

Fig. 1. Intensity distributions in focal region for $m=1$, NA = 0.7 and n ranges from (a) $n=2$, (b) $n=3$, (c) $n=4$, (d) $n=5$, respectively.