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Scattering of plane waves by a wedge with different face impedances

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ABSTRACT

The scattering process of plane waves by a wedge with different face impedances is examined in terms of the closed form series solution. A new boundary condition is derived using the solution of the reflection problem of plane waves by an impedance plane. The series solution is obtained for the wedge problem. The results are investigated numerically.

1. Introduction

When the waves meet an obstacle on their path of propagation, they bend and enter the places in the shadow regions. This phenomenon is named as diffraction in the literature [1]. The portion of the waves that propagates without unaffected by the obstacle is the geometrical optics (GO) field. The sum of the diffracted waves and (GO) fields is known as the scattered waves. The rigorous way of obtaining the solution of a scattering problem is the method of separation of variables [2]. The availability of the exact solutions of the Helmholtz equation with this method is restricted with the geometry of the scatterer. Only the solutions of some canonical problems were obtained [3]. The scattering problems by complex bodies are investigated by high frequency asymptotic methods [4].

Another important factor in the examination of a scattering problem is the boundary conditions that must be satisfied by the total field on the scattering surface. There are two limiting cases for the boundary conditions. A surface can reflect all the waves that hit on it or absorb them. Two boundary conditions can be defined for a perfectly reflecting surface. The total field can be equal to zero (Dirichlet condition) or its normal derivative is equal to zero (Neumann condition) on the surface. There are no rigorous boundary conditions for a perfectly absorbing surface. In practical, the surface absorbs some of the incident radiation and reflects a portion which is decreased in amplitude. A dielectric coated metallic surface is an example of this case [5,6]. Such surfaces are modeled by the impedance boundary conditions, which define a ratio of the field and its normal derivative on the scattering surface [7].

The first investigation of the scattering problem of waves by an impedance half-plane was put forward by Raman and Krishnan [8]. They realized that the solution of the scattering problem of waves by a perfectly conducting half-plane was not adequate for the physical interpretations of the observations on edge diffraction. For this reason they examined the case for a non-perfectly conducting half-screen. They considered the solution of Sommerfeld, which is expressed in terms of Fresnel integrals [9], and introduced the scattered fields by a non-perfectly conducting half-plane by multiplying the expression of the reflected scattered waves by a suitable complex reflection coefficient. In 1952, Senior obtained the solution of the half-screen problem for equal face impedances by solving an integral equation with the method of Wiener–Hopf factorization [10]. The solution of the problem was derived for the two cases of electric and magnetic polarization. Malyuzhinets takes into account a more general problem of scattering of waves by a wedge with two impedance faces and obtained the solution using the complex integration technique of Sommerfeld [11,12]. In the same years, Senior extended his research on the diffraction of waves by an impedance wedge and scattering by a half-plane for oblique incidence [13,14]. He used the same method of Wiener–Hopf factorization. The diffraction problem of waves by the impedance half-screen and wedge was also investigated by Williams, who used a contour integration method in order to obtain the solution [15,16]. The following studies were focused on the physical interpretations and uniform expressions of the former results [17–21]. However the developed expressions for the diffracted waves are not suitable for practical applications since they include a meromorphic function, named the Malyuzhinets function. An improved physical optics approach was put forward for us for the scattering problems of impedance half-screen and wedge [22,23].

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It is the aim of this study is to obtain the closed form series solution of the scattering problem of plane waves by a wedge with different face impedances. First of all we will investigate the problem of diffraction of waves by an interface between two impedance half-planes. The results will be compared with that of Malyuzhinets and the limiting cases of soft and hard half-planes will also be examined. The solution of the wedge problem will be obtained directly from the expressions of the previous problem. The result will be plotted numerically.

A time factor of $\exp(jwt)$ is suppressed throughout the paper. w is the angular frequency.

2. Diffraction by an interface between two impedance half-screens

An interface between two impedance half-planes is taken into account. An incident plane wave of

$$u_i = u_0 e^{jk\rho\cos(\phi-\phi_0)} \tag{1}$$

is illuminating the plane. k is the wave-number which is equal to $2\pi/\lambda$ where λ is the wave-length. ϕ_0 is the angle of incidence. The polar coordinates are defined as (ρ, ϕ). The geometry of the problem is given in Fig. 1.

$Z_{1,2}$ is the impedance of the surface. P is the observation point. We will define the expression of

$$\sin\theta_{1,2} = \frac{Z_0}{Z_{1,2}} \tag{2}$$

where Z_0 is the impedance of the free space. The impedance boundary condition can be given by the equation of $u|_S = (1/jk\sin\theta)(\partial u/\partial n)_S$. Now we will consider a whole impedance plane, the surface impedance of which is equal to Z. The scattered fields can be immediately written as

$$u_t = u_i + \frac{\sin\phi_0 - \sin\theta}{\sin\phi_0 + \sin\theta} u_r \tag{3}$$

for u_r is the reflected field. We propose the boundary conditions of

$$(u_i - u_r)|_S = 0 \tag{4}$$

and

$$\left.\frac{\partial(u_i + u_r)}{\partial n}\right|_S = 0 \tag{5}$$

for soft and hard surfaces, respectively. n is the unit normal vector of the surface. Then Eq. (3) can be rewritten as

$$u_t = \frac{\sin\theta}{\sin\phi_0 + \sin\theta} u_s + \frac{\sin\phi_0}{\sin\phi_0 + \sin\theta} u_h \tag{6}$$

where u_s and u_h represent the scattered fields by soft and hard surfaces, respectively. Eq. (6) shows that the scattered fields by an impedance surface can be expressed in terms of the scattered fields by soft and hard surfaces with the same geometry. Note that the scattered field, given by Eq. (6), reduced to the scattered field by soft and hard surfaces for $\sin\theta \to \infty$ and $\sin\theta \to 0$, respectively. If $\sin\theta$ is equal to $\sin\phi_0$, the condition of a black surface can be obtained [24].

Now we return to the problem, given in Fig. 1. There are two limiting cases. If Z_2 is equal to zero, the surface at $\phi = \pi$ will be a soft surface. If Z_2 approaches to infinity, then the surface becomes a hard surface. In fact these cases are the two limiting values of the reflection coefficient, given in Eq. (3). In the first and second cases R is equal to -1 and 1, respectively. Note that R is defined by the equation of

$$R = \frac{\sin\phi_0 - \sin\theta}{\sin\phi_0 + \sin\theta} \tag{7}$$

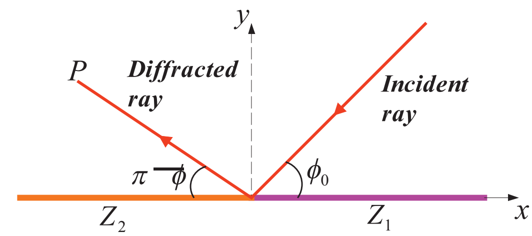

Fig. 1. Geometry of the interface.

We will introduce the notions of α and β, which are equal to

$$\alpha = \frac{\sin\theta}{\sin\phi_0 + \sin\theta} \tag{8}$$

and

$$\beta = \frac{\sin\phi_0}{\sin\phi_0 + \sin\theta} \tag{9}$$

respectively.

2.1. Case 1

First of all we will consider the scattering problem for $Z_2 = 0$. Note that $\alpha_2 = 1$ and $\beta_2 = 0$ for this case. The Helmholtz equation of

$$\nabla^2 u + k^2 u = 0 \tag{10}$$

will be solved with the boundary condition of

$$u_t|_{\phi=\pi} = 0 \tag{11}$$

where u_t is equal to $\alpha_1 u_s + \beta_1 u_h$ according to Eq. (6). A general solution of the Helmholtz equation can be given by the expression of

$$u = J_\nu(k\rho)[A_\nu \sin(\nu\phi) + B_\nu \cos(\nu\phi)] \tag{12}$$

in the polar coordinates for ν is the separation constant. A_ν and B_ν are the constant coefficients. $J_n(x)$ is the Bessel function. Since the problem includes the origin, the Neumann function is not taken into account. The scattered field can be written as

$$u_t = J_\nu(k\rho)[\alpha_1 A_\nu \sin(\nu\phi) + \beta_1 B_\nu \cos(\nu\phi)] \tag{13}$$

according to the soft and hard boundary conditions on the plane of $\phi = 0$. The total scattered field is found to be

$$u_t = 4u_0 \left[\alpha_1 \sum_{n=1}^{\infty} J_n(k\rho) e^{jn(\pi/2)} \sin(n\phi)\sin(n\phi_0) + \beta_1 \sum_{n=0}^{\infty} J_{\vartheta_n}(k\rho) e^{j\vartheta_n(\pi/2)} \cos(\vartheta_n\phi)\cos(\vartheta_n\phi_0)\right] \tag{14}$$

when the boundary condition, in Eq. (11), is taken into account. ϑ_n is equal to $(2n+1)/2$. The condition of $\alpha_1 < \alpha_2$ is valid for this situation. The coefficients of A_ν and B_ν are determined from the series expression of the incident field [25].

2.2. Case 2

Z_2 approaches to infinity. α_2 and β_2 are equal to zero and one, respectively. The half-plane at $\phi = \pi$ is a hard surface and the boundary condition can be given by the expression of

$$\left.\frac{\partial u_t}{\partial n}\right|_{\phi=\pi} = 0 \tag{15}$$