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Nonparaxial Lorentz and Lorentz–Gauss beams

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Abstract

The Lorentz and Lorentz–Gauss beams are extended to the nonparaxial regime. Analytical propagation expressions of nonparaxial Lorentz and Lorentz–Gauss beams in free space are derived, and the propagation of paraxial Lorentz and Lorentz–Gauss beams is treated as a special case of our general results. The propagation properties of Lorentz and Lorentz–Gauss beams are illustrated and compared with numerical examples.

Keywords: Nonparaxial Lorentz and Lorentz–Gauss beams; Rayleigh–Sommerfeld diffraction integral; Propagation

1. Introduction

As yet, the propagation of laser beams has been extensively studied within the framework of the paraxial approximation [1]. In recent years a variety of new types of beams, such as Ince–Gaussian, Helmholtz–Gauss, Laplace–Gauss, Mathieu–Gauss, Lorentz, Lorentz–Gauss, super-Lorentzian beams, etc. have been introduced [2–7]. Among them the Lorentz and Lorentz–Gauss beams are of particular interest, because as shown in Refs. [8,9], for certain laser sources, e.g., for double-heterojunction $Ga_{1-x}Al_xAs$ lasers, a truncated Lorentzian distribution is a better approximation in the direction perpendicular to the junction than a Gaussian one. However, for $Ga_{1-x}Al_xAs$ lasers the active regions are as narrow as 0.1 μm for a typical emission wavelength of ~0.8 μm. Therefore, for such a case, i.e., for beams with small spot size and large divergence angle the paraxial approximation is no longer valid [10,11]. The purpose of the present paper is to extend the Lorentz and Lorentz–Gauss beams introduced in Ref. [6] to the nonparaxial regime. Based on the Rayleigh–Sommerfeld diffraction integral, closed-form propagation expressions of nonparaxial Lorentz and Lorentz–Gauss beams in free space are derived and some interesting special cases are discussed. Specifically, our results reduce to those in Ref. [6] in the paraxial approximation. The propagation properties of Lorentz and Lorentz–Gauss are illustrated and compared.

2. Propagation of nonparaxial Lorentz beams in free space

Assume that the field of a Lorentz beam at the place $z=0$ takes the form [6]

$$E_L(x_0,y_0,0)=\frac{A_L}{w_xw_y}\frac{1}{[1+(x_0/w_x)^2]}\frac{1}{[1+(y_0/w_y)^2]},\qquad(1)$$

where w_x and w_y are parameters related to the beam width in the x and y directions, respectively, and A_L is a constant.

The beam propagation for the general case is characterized by the Rayleigh–Sommerfeld diffraction

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integral of the form [12]

$$E(x,y,z) = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E(x_0,y_0,0)\frac{\partial}{\partial z}\left[\frac{\exp(ikR)}{R}\right] dx_0\, dy_0, \tag{2}$$

where $R = |\mathbf{r}-\mathbf{r}_0| = \sqrt{(x-x_0)^2+(y-y_0)^2+z^2}$, $\mathbf{r}_0 = x_0\mathbf{i}+y_0\mathbf{j}$, $\mathbf{r} = x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$, $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ being unit vectors in the x, y and z directions, respectively, and k denotes the wave number related to the wavelength λ by $k = 2\pi/\lambda$.

Expanding R into

$$R = |\mathbf{r}-\mathbf{r}_0| \approx r + \frac{x_0^2+y_0^2-2xx_0-2yy_0}{2r}, \tag{3}$$

with $r = \sqrt{x^2+y^2+z^2}$, and substituting Eqs. (1) and (3) into Eq. (2), by means of the convolution theorem in Fourier transform [13], tedious but straightforward integral calculations yield

$$E_L(x,y,z) = \frac{A_L\pi^2}{4}\frac{z}{i\lambda r}\frac{\exp(ikr)}{r}\exp\left[-\frac{ik}{2r}(x^2+y^2)\right] \times (V_{x,L}^{+}+V_{x,L}^{-})(V_{y,L}^{+}+V_{y,L}^{-}), \tag{4}$$

where

$$f_\eta = \frac{1}{kw_\eta} \quad (f_\eta\text{ -parameter}), \tag{5}$$

$$V_{\eta,L}^{\pm} = \exp\left[\frac{1}{2ikr}\left(\frac{1}{f_\eta}\pm ik\eta\right)^2\right] \times\left\{1-\mathrm{erf}\left[\sqrt{\frac{1}{2ikr}}\left(\frac{1}{f_\eta}\pm ik\eta\right)\right]\right\}, \tag{6}$$

$\eta = x, y$ (unless otherwise stated), and $\mathrm{erf}(\cdot)$ denotes the error function. Eq. (4) provides the analytical free-space propagation expression of Lorentz beams beyond the paraxial approximation, which is valid for $R \gg \lambda$ and for both the Fresnel zone and Fraunhofer zone, and indicates that $E_L(x,\ y,\ z)$ depends on the f_x- and f_y-parameters and observation position. Some special cases of Eq. (4) are of interest.

(1) On-axis field
On placing $x = y = 0$ into Eq. (4), the on-axis field is given by

$$E_L(0,0,z) = \frac{A_L\pi^2}{i\lambda}\frac{\exp(ikz)}{z}\exp\left(\frac{1}{2ikf_x^2 z}\right) \times\left[1-\mathrm{erf}\left(\frac{1}{f_x}\sqrt{\frac{1}{2ikz}}\right)\right] \times\exp\left(\frac{1}{2ikf_y^2 z}\right)\left[1-\mathrm{erf}\left(\frac{1}{f_y}\sqrt{\frac{1}{2ikz}}\right)\right]. \tag{7}$$

(2) Far-field expression
Under the far-field approximation

$$R \approx r - \frac{xx_0+yy_0}{r}. \tag{8}$$

Eq. (4) reduces to

$$E_{L,f}(x,y,z) = \frac{A_{L,f}\pi^2 z}{i\lambda r}\frac{\exp(ikr)}{r}\exp\left[-\frac{1}{r}\left(\frac{|x|}{f_x}+\frac{|y|}{f_y}\right)\right]. \tag{9}$$

Furthermore, from Eq. (9) the on-axis intensity in the far field is given by

$$I_{L,f}(0,0,z) = |E_{L,f}(0,0,z)|^2 = \frac{A_{L,f}^2\pi^4}{\lambda^2 z^2}. \tag{10}$$

Eq. (10) implies that the on-axis intensity in the far field is inversely proportional to z^2.

(3) Paraxial propagation
If the paraxial approximation

$$r \approx z + \frac{x^2+y^2}{2z} \tag{11}$$

is used, Eq. (4) simplifies to

$$E_{L,p}(x,y,z) = \frac{A_{L,p}\pi^2}{4}\frac{\exp(ikz)}{i\lambda z} \times (V_{x,Lp}^{+}+V_{x,Lp}^{-})(V_{y,Lp}^{+}+V_{y,Lp}^{-}), \tag{12}$$

where

$$V_{\eta,Lp}^{\pm} = \exp\left[\frac{1}{2ikz}\left(\frac{1}{f_\eta}\pm ik\eta\right)^2\right] \times\left\{1-\mathrm{erf}\left[\sqrt{\frac{1}{2ikz}}\left(\frac{1}{f_\eta}\pm ik\eta\right)\right]\right\}. \tag{13}$$

Eq. (12) is consistent with Eq. (15) in Ref. [6].

Figs. 1(a) and (b) give normalized intensity distributions $I(x,\ 0,\ z)/I(0,\ 0,\ z)$ of a Lorentz beam with $f_x = f_y = f = 0.83$ at the planes (a) $z = 6\,\mu\mathrm{m}$ and (b) $z = 100\,\mu\mathrm{m}$, respectively. For comparison the paraxial results by using Eq. (12) are plotted together. In Fig. 1(b) the use of Eqs. (4) and (12) delivers the same result. As can been seen, the intensity profiles in the Fresnel zone in Fig. 1(a) and in the Fraunhofer zone in Fig. 1(b) are somewhat different, although both are bell shaped. In addition, a noticeable difference between the paraxial and nonparaxial results appears when the f_x-parameter is large enough, namely, $w_x \leqslant \lambda$. However, as shown in Figs. 2(a) and (b), with decreasing f_x-parameter form $f_x = 0.62$ to $f_x = 0.23$, the paraxial approximation is applicable, which can be illustrated in more detail in Fig. 3, where the relative difference between the paraxial and nonparaxial intensities