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Propagation properties of a partially polarized electromagnetic twist anisotropic Gaussian Schell-model beam in turbulent atmosphere

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Abstract

Based on the extended Huygens–Fresnel integral formula, analytical formulae for the elements of cross-spectral density matrix of partially polarized electromagnetic twist anisotropic Gaussian Schell-model (TAGSM) beam propagating in turbulent atmosphere can been derived by a tensor method. Our main attention was focus on the effect of the atmospheric turbulence, twist parameters and partial coherence on the spectral degree of polarization, the spectral degree of coherence and the spectral density. Numerical calculation results and analysis are given.

Keywords: Twist anisotropic Gaussian Schell-model (TAGSM) beams; Turbulent atmosphere; Tensor method; Spectral properties

1. Introduction

Recently, increasing attentions have been devoted toward both partially coherent and partially polarized light beams propagating in turbulent atmosphere due to their importance in coherence theory and practical applications [1–10]. After the unified theory of coherence and polarization had been presented by Wolf [11], it became possible to determine the properties of light beams, in particular, in the spectral density, in the spectral degree of coherence and in the polarization properties. All the above-mentioned properties of beams can be determined from the basic quantity of the unified theory, i.e. from their 2×2 electric correlation matrix, also know as the cross-spectral density matrix [12,13]. Using the unified theory of coherence and polarization, changes in the degree of polarization of Gaussian Schell-model (GSM) beams propagating in turbulent atmosphere have been investigated [5–8]. To the best of our knowledge, changes in the spectral degree of polarization, spectral degree of coherence and spectral density of the partially polarized electromagnetic twist anisotropic Gaussian Schell-model (TAGSM) beams in turbulent atmosphere have never been studied.

On the other hand, the conventional method to tread GSM beams is the Wigner distribution function [14]. Recently, Lin and coworkers [15–18] introduced a new tensor method instead of Wigner distribution function to treat the propagation of partially coherent GSM beams. This method proves to be a powerful and convenient way to treat the propagation of partially coherent and partially polarized GSM beams through complex optical systems and dispersive media.

In this paper, based on the extended Huygens–Fresnel principle, a general analytic formulae for the elements of cross-spectral density matrix of partially polarized electromagnetic TAGSM beams propagating in turbulent atmosphere can been derived by the tensor method. The influence of the atmospheric turbulence, twist

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parameters and partial coherence on the spectral degree of polarization, the spectral degree of coherence and the spectral density are investigated in detail. Numerical results are given in Section 3.

2. Analysis of partially polarized electromagnetic TAGSM beams propagating in turbulent atmosphere in terms of tensor method

In this section, we will outline briefly the tensor method for electromagnetic GSM beams. The cross-spectral density matrix [7] of the beam at $z=0$ can be expressed in the following tensor form [5]:

$$W_{sij}(\tilde{r},0;\omega)=A_iA_jB_{ij}\exp\left(-\frac{ik}{2}\tilde{r}^TM_{sij}^{-1}\tilde{r}\right)\quad(i,j=x,y) \tag{1}$$

Here $\tilde{r}^T=(r_1^T,r_2^T)$, r_1 and r_2 are the position vectors of two arbitrary points in the transverse plane, $k=2\pi/\lambda=\omega/c$ denotes the wave number with λ being the wavelength and c is the wave speed in vacuum. M_{sij}^{-1} is a transpose symmetric matrix, and it is called partially coherent complex curvature tensor. For electromagnetic TAGSM beams, the corresponding partially coherent complex curvature tensor M_{sij}^{-1} takes the following form [19]:

$$M_{sij}^{-1}=\begin{pmatrix} R_{sij}^{-1}+\left(-\frac{i}{2k}\right)(\sigma_{Isij}^2)^{-1}-\frac{i}{k}(\sigma_{gsij}^2)^{-1} & \frac{i}{k}(\sigma_{gsij}^2)^{-1}+u_{sij}J \\ \frac{i}{k}(\sigma_{gsij}^2)^{-1}+u_{sij}J & -R_{sij}^{-1}+\left(-\frac{i}{2k}\right)(\sigma_{Isij}^2)^{-1}-\frac{i}{k}(\sigma_{gsij}^2)^{-1}\end{pmatrix} \tag{2}$$

Here u_{sij} is a real-valued constant named the twist factor and $J=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$. σ_{Is}^2 stands for transverse spot width matrix, σ_{gs}^2 denotes transverse coherence width matrix. σ_{Is}^2 and σ_{gs}^2 are all 2×2 matrixes with transpose symmetry, given by

$$(\sigma_{Is}^2)^{-1}=\begin{pmatrix}\sigma_{I11}^{-2} & \sigma_{I12}^{-2}\\ \sigma_{I21}^{-2} & \sigma_{I22}^{-2}\end{pmatrix}\quad(\sigma_{gs}^2)^{-1}=\begin{pmatrix}\sigma_{g11}^{-2} & \sigma_{g12}^{-2}\\ \sigma_{g21}^{-2} & \sigma_{g22}^{-2}\end{pmatrix} \tag{3}$$

The coefficients A_i, A_j, and B_{ij} are independent of position but may depend on frequency [5]. Moreover, the factor B_{ij} has the properties [20]

$$B_{ij}=1\ \text{when}\ i=j;\quad |B_{ij}|\leqslant 1\ \text{when}\ i\neq j,\ B_{ij}=B_{ij}^*$$

Based on the extended Huygens–Fresnel integral, we can obtain the following propagation formula for the cross-spectral density matrix of electromagnetic TAGSM beams through a turbulent atmosphere [21]:

$$W_{ij}(\tilde{\rho},z;\omega)=\frac{k^2}{4\pi^2z^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}W_{sij}(\tilde{r},0;\omega)\times\exp\left[-\frac{ik}{2z}(r_1-\rho_1)^2+\frac{ik}{2z}(r_2-\rho_2)^2\right]\times\langle\exp[\psi^*(r_1,\rho_1,z;\omega)+\psi(r_2,\rho_2,z;\omega)]\rangle\,dr_1\,dr_2 \tag{4}$$

Here $\tilde{\rho}^T=(\rho_1^T,\rho_2^T)$, ρ_1 and ρ_2 are position vectors of two arbitrary points in the output plane. $\langle\ \rangle$ denotes averaging over the ensemble of turbulent media and can be expressed as [22,23]:

$$\langle\exp[\psi^*(r_1,\rho_1,z;\omega)+\psi(r_2,\rho_2,z;\omega)]\rangle=\exp\left\{-\frac{[(r_1-r_2)^2+(r_1-r_2)(\rho_1-\rho_2)+(\rho_1-\rho_2)^2]}{\rho_0^2}\right\} \tag{5}$$

Here $\rho_0=(0.545C_n{}^2k^2z)^{-3/5}$ is the coherence length of a wave propagating in the turbulent atmosphere whose behavior is described by the Kolmogorov model and C_n^2 denotes the refractive index structure constant denoting the strength of turbulent atmosphere. In the derivation of Eq. (5), we have quadratic approximation for Rytov's phase structure function in order to obtain simpler and viewable analytical results [22,23]. This quadratic approximation has been shown to be reliable and has been widely investigated [24–27].

After some rearrangement, we can express Eq. (4) in the following tensor form:

$$W_{ij}(\tilde{\rho},z;\omega)=\frac{k^2}{4\pi^2[\det(\tilde{B})]^{1/2}}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}W_{sij}(\tilde{r},0;\omega)\times\exp\left[-\frac{ik}{2}(\tilde{r}^T\tilde{B}^{-1}\tilde{r}-2\tilde{r}^T\tilde{B}^{-1}\tilde{\rho}+\tilde{\rho}^T\tilde{B}^{-1}\tilde{\rho}\right]\times\exp\left[-\frac{ik}{2}\tilde{r}^T\tilde{P}\tilde{r}-\frac{ik}{2}\tilde{r}^T\tilde{P}\tilde{\rho}-\frac{ik}{2}\tilde{\rho}^T\tilde{P}\tilde{\rho}\right]d\tilde{r} \tag{6}$$

Here $\tilde{B}=\begin{pmatrix}zI & 0\\ 0 & -zI\end{pmatrix}$, $\tilde{P}=\frac{2}{ik\rho_0^2}\begin{pmatrix}I & -I\\ -I & I\end{pmatrix}$ and I is a 2×2 matrix.

Substituting Eq. (1) into Eq. (6), we obtain the following expression for the cross-spectral density matrix (after some vector integration and tensor operation):

$$W_{ij}(\tilde{\rho},z;\omega)=A_iA_jB_{ij}(\det[\tilde{I}+\tilde{B}(M_{sij}^{-1}+\tilde{P})])^{-1/2}\times\exp\left[-\frac{ik}{2}\tilde{\rho}^TM_{ij}^{-1}\tilde{\rho}\right] \tag{7}$$