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Exhaustive study of reflection and refraction at spherical surfaces on the basis of the newly discovered generalized vectorial laws of reflection and refraction

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ABSTRACT

This paper deals with the application of the newly discovered generalized vectorial laws of reflection and refraction for making an extensive analysis of reflection and refraction at spherical surfaces. Various cases of image formation have been considered. It has been observed that, direct application of the generalized vectorial laws of reflection and refraction leads to the appropriate formula of ray optics for each of the cases considered. The analysis of the results are ultimately summed up to give birth to the generalized mirror formula as well as the generalized formula for refraction at spherical surfaces. Also by using the results obtained for refraction at spherical surfaces, the Lens makers' formulae have been finally offered.

1. Introduction

This paper is concerned with the application of the generalized vectorial laws of reflection and refraction [1] discovered by the author in 2005. In [2], theoretical proof of the said discovered laws has been offered on the basis of the principle of conservation of momentum of photon. In [3], the said discovered laws have been employed to develop a lot of interesting physical insights to the mirror rotation problem as well as to the problem of rotation of refracting surface. The most remarkable fact that has been discovered in the said study is that the proposition "velocity of light is unbeatable" is not correct. Rather it is possible to have velocity exceeding the velocity of light – a result not in agreement with the Special theory of relativity.

In order to test the efficiency of the generalized vectorial laws of reflection and refraction, an extensive use of the said laws has been made in this paper to deal with the cases of reflection and refraction at spherical surfaces considering various kinds of image formation. The overall study reveals that the various formulae of ray optics with special reference to reflection and refraction at spherical surfaces [4–10] could be achieved by direct application of the newly discovered generalized vectorial laws of reflection and refraction. The results are summed up by offering a generalized mirror formula and a generalized formula for refraction at spherical surfaces. Finally the derivation of the Lens makers' formulae has also been considered on the basis of the formulae developed for refraction at spherical surfaces.

It may be mentioned here that only spherical mirror (refracting surface) of comparatively small aperture will be considered in this paper. Furthermore, on account of space limitation and in order to enhance the readability of the paper, treatments are kept restricted to limited number of cases of image formation. Following similar procedures, interested readers may derive relevant formula for each of the remaining cases of image formation.

Notations:

i: unit vector along the direction of incident ray.
r: unit vector along the direction of reflected ray.
R: unit vector along the direction of refracted ray.
n: unit vector along the conventional direction of the surface area of the reflecting or refracting surface at the point of incidence.
I and **J**: rectangular unit vectors.
R: radius of curvature of spherical mirror (refracting surface).

2. The discovered laws

In this section the laws reported in [1] are being presented.

The generalized vectorial law of reflection:

If **i** and **r** represent unit vectors along the directions of the incident ray and reflected ray respectively and if **n** represents unit

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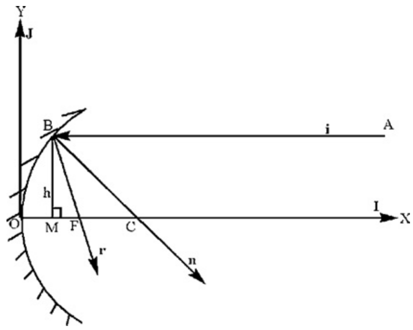

Fig. 1. Diagram showing typical reflection in a concave mirror.

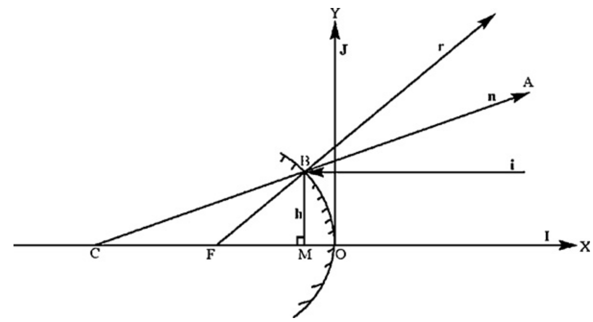

Fig. 2. Diagram showing typical reflection in a convex mirror.

vector along the direction of the positive unit normal to the reflector at the point of incidence, then $\mathbf{n} \times \mathbf{i} = \mathbf{n} \times \mathbf{r}$.

The generalized vectorial law of refraction:

If $\mathbf{i}$ and $\mathbf{R}$ represent unit vectors along the directions of the incident ray and refracted ray of particular colour respectively and if $\mathbf{n}$ represents unit vector along the direction of the positive unit normal to the surface of separation at the point of incidence, then $\mathbf{n} \times \mathbf{i} = \mu(\mathbf{n} \times \mathbf{R})$, where μ = refractive index of the second medium with respect to the first medium for the particular colour of light under consideration.

3. Application of the generalized vectorial law of reflection to the cases of spherial mirrors

3.1. To obtain the relationship between focal length and radius of curvature of a spherical mirror

Case 1: For concave mirror

It clearly follows from Fig. 1 that in this case, $\mathbf{i} = -\mathbf{I}$, $\mathbf{n} = (\mathbf{BM} + \mathbf{MC})/I\ \mathbf{BM} + \mathbf{MC}\ I = (-h\mathbf{J} + R\mathbf{I})/\sqrt{(h^2 + R^2)}$, for mirror of small aperture and $\mathbf{r} = (\mathbf{BM} + \mathbf{MF})/I\ \ \mathbf{BM} + \mathbf{MF}\ \ I = (-h\mathbf{J} + f\mathbf{I})/\sqrt{(h^2 + f^2)}$, for mirror of small aperture.

Then from the generalized vectorial law of reflection of light we have,

$$\mathbf{n} \times \mathbf{i} = \mathbf{n} \times \mathbf{r}$$

or, $-h\mathbf{k}/\sqrt{(h^2 + R^2)} = (hf\mathbf{k} - Rh\mathbf{k})/(\sqrt{(h^2 + R^2)}(\sqrt{(h^2 + f^2)})$; or, $1/\sqrt{(h^2 + R^2)} = (R - f)/(\sqrt{(h^2 + R^2)}(\sqrt{(h^2 + f^2)})$; or, $1/R = (R - f)/Rf$, since for mirror of small aperture, h is small and hence h^2 will be still smaller and can therefore be neglected; or, $f = R/2$, which is the required relation.

Case 2: For convex mirror

It can be readily seen from Fig. 2 that in this case, $\mathbf{i} = -\mathbf{I}$, $\mathbf{n} = (\mathbf{CM} + \mathbf{MB})/I\ \mathbf{CM} + \mathbf{MB}\ I = (R\mathbf{I} + h\mathbf{J})/\sqrt{(h^2 + R^2)}$, for mirror of small aperture and $\mathbf{r} = (\mathbf{FM} + \mathbf{MB})/I\ \mathbf{FM} + \mathbf{MB}\ I = (f\mathbf{I} + h\mathbf{J})/\sqrt{(h^2 + f^2)}$, for mirror of small aperture.

Then from the generalized vectorial law of reflection it follows that

$$\frac{h\mathbf{k}}{\sqrt{(h^2 + R^2)}} = \frac{(Rh\mathbf{k} - hf\mathbf{k})}{(\sqrt{(h^2 + R^2)}(\sqrt{(h^2 + f^2)})}$$

or, $1/\sqrt{(h^2 + R^2)} = (R - f)/(\sqrt{(h^2 + R^2)}(\sqrt{(h^2 + f^2)})$; or, $1/R = (R - f)/Rf$, since for mirror of small aperture, h is small and hence h^2 will be still smaller and can therefore be neglected; or, $f = R/2$, which is the required relation.

3.2. To obtain relation between object distance, image distance and focal length

Case 1: For convex mirror

When object is real and image is virtual:

Considering Fig. 3, we have, $\mathbf{n} = \mathbf{CL}/I\mathbf{CL}I = (R\mathbf{I} + h\mathbf{J})/\sqrt{(h^2 + R^2)} = (2f\mathbf{I} + h\mathbf{J})/\sqrt{(4f^2 + h^2)}$, since for spherical mirror of small aperture, $f = R/2$; or, $\mathbf{n} = (2f\mathbf{I} + h\mathbf{J})/2f$, since for a spherical mirror of small aperture, h being small, h^2 would be still smaller and can be neglected.

Also, $\mathbf{i} = \mathbf{OL}/I\mathbf{OL}I = (-u\mathbf{I} + h\mathbf{J})/\sqrt{(u^2 + h^2)} = (-u\mathbf{I} + h\mathbf{J})/u$, since for a spherical mirror of small aperture, h^2 can be neglected.

Again, $\mathbf{r} = \mathbf{IL}/I\mathbf{IL}I = (v\mathbf{I} + h\mathbf{J})/\sqrt{(v^2 + h^2)} = (v\mathbf{I} + h\mathbf{J})/v$, since for a spherical mirror of small aperture, h^2 can be neglected.

Now the generalized vectorial law of reflection of light demands that $\mathbf{n} \times \mathbf{I} = \mathbf{n} \times \mathbf{r}$ and hence in this case we have,

$$\frac{(2f\mathbf{I} + h\mathbf{J})}{2f} \times \frac{(-u\mathbf{I} + h\mathbf{J})}{u} = \frac{(2f\mathbf{I} + h\mathbf{J})}{2f} \times \frac{(v\mathbf{I} + h\mathbf{J})}{v}$$

or, $h/u + h/2f = h/v - h/2f$; or,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad (1)$$

which is the required relation.

When object is virtual and image is real:

Considering Fig. 4, we have, $\mathbf{n} = \mathbf{CL}/I\mathbf{CL}I = (R\mathbf{I} + h\mathbf{J})/\sqrt{(h^2 + R^2)} = (2f\mathbf{I} + h\mathbf{J})/\sqrt{(4f^2 + h^2)}$, since for spherical mirror of small aperture, $f = R/2$; or, $\mathbf{n} = (2f\mathbf{I} + h\mathbf{J})/2f$, since for a spherical mirror of small aperture, h being small, h^2 would be still smaller and can be neglected.

Also, $\mathbf{i} = \mathbf{LO}/I\mathbf{LO}I = (-u\mathbf{I} - h\mathbf{J})/\sqrt{(u^2 + h^2)} = (-u\mathbf{I} - h\mathbf{J})/u$, since for a spherical mirror of small aperture, h^2 can be neglected.

Again, $\mathbf{r} = \mathbf{LI}/I\mathbf{LI}I = (v\mathbf{I} - h\mathbf{J})/\sqrt{(v^2 + h^2)} = (v\mathbf{I} - h\mathbf{J})/v$, since for a spherical mirror of small aperture, h^2 can be neglected.

It, therefore, follows from the generalized vectorial law of reflection that in this case we have,

$$\frac{(2f\mathbf{I} + h\mathbf{J})}{2f} \times \frac{(-u\mathbf{I} - h\mathbf{J})}{u} = \frac{(2f\mathbf{I} + h\mathbf{J})}{2f} \times \frac{(v\mathbf{I} - h\mathbf{J})}{v}$$

or,

$$\frac{1}{v} - \frac{1}{u} = \frac{-1}{f}, \quad (2)$$

which is the required relation.

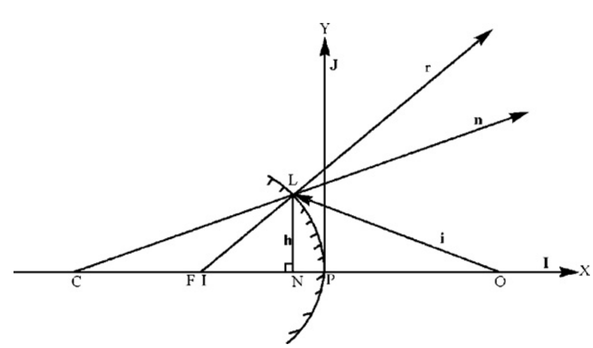

Fig. 3. Diagram showing formation of virtual image of a real object by a convex mirror.