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Fringe waves in wedge diffraction

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ABSTRACT

Explicit expressions for the non-uniform currents of the physical theory of diffraction are derived in terms of Fresnel functions for wedge diffraction by taking into account the surface integrals of the modified theory of physical optics. The obtained fringe waves are compared numerically by the asymptotic representations, found in the literature.

1. Introduction

Physical optics (PO) is an integrative high frequency technique that is widely used in the applications of the optics and electromagnetics [1–4]. The surface current on a scatterer is approximated with an equivalent current that flows on a tangential surface on the point of scattering [5]. Thus the method of PO gives more accurate results as the frequency of the incoming wave increases or the dimensions of the scatterer is larger than the wave-length of the wave. Although PO leads to the exact geometrical optics (GO) fields, the edge point contribution of the integral at the discontinuities of the scattering object yields incorrect diffracted waves. An improvement to this method was proposed by Ufimtsev, in order to obtain the exact edge diffracted wave for complex scattering problems [6]. He defined the equivalent currents of PO as the uniform current component since they are evaluated for an infinite tangential surface. Ufimtsev thought that the error of the PO contribution was because of the sudden discontinuity that occurs on the path of the uniform currents. Thus he suggested adding an additional component to the currents of PO, which are named as fringe or non-uniform currents [6,7]. The expressions for the fringe currents are obtained by subtracting the asymptotic diffracted waves of PO from the geometrical theory of diffraction (GTD) fields that are found from the rigorous solution of Sommerfeld [8,9]. Since the two asymptotic expressions of the diffracted fields are non-uniform (approaches to infinity at the transition regions), the resultant field is uniform everywhere. This is the result of $\infty - \infty$ indeterminacy at the transition zones. The physical theory of diffraction (PTD) has important applications in electromagnetic scattering problems and also more practical areas as the development of the B-2 aircraft [7,9]. The fringe currents, used in the literature, are the asymptotic expressions which are valid for the large argument condition of the Fresnel function [10].

It is the aim of this paper is to obtain the explicit expressions of the fringe waves in terms of Fresnel integrals for wedge diffraction by using the surface integrals of the modified theory of physical optics (MTPO), which are derived in order to evaluate the exact diffracted waves for perfectly conducting (PEC) half-planes and wedge [1,2]. Two cases of soft and hard surfaces will be taken into account. The fringe waves will be expressed in terms of Fresnel integrals. Since the integrals of PO and MTPO have similar constructions, there is no need to find the asymptotic expansion of the diffracted waves. For this reason the obtained fringe waves will be more accurate than the asymptotic ones. It may seem to be trivial to deal with the fringe currents after MTPO since MTPO directly gives the exact diffracted waves for PEC structures without the evaluation of additional current components. There are two reasons for such a study. The first one is the physical effects of the fringe currents like depolarization [6,13]. The second reason is the applications that used PTD in the literature. After the evaluation of more compact expressions for the fringe fields, these studies must be revised. The new representations of the fringe waves will be compared by the asymptotic ones, numerically and their regions of harmony will be investigated.

A time factor of exp(jwt) is considered and suppressed throughout the paper.

2. Evaluation of the fringe waves

According to PTD, fringe currents are induced in the neighborhood of the diffracting edge and their amplitude decreases to zero away from the edge discontinuity. In the context of PTD the expres-

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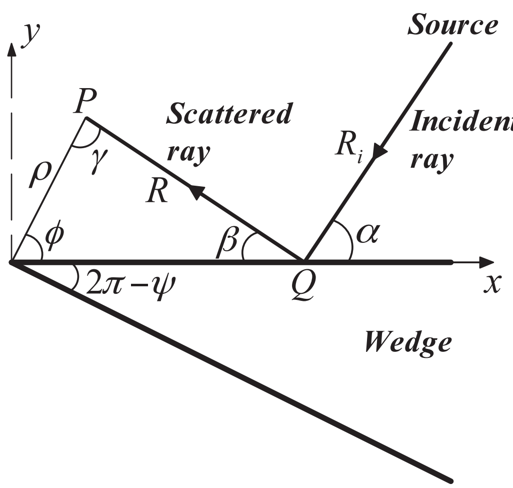

Fig. 1. Geometry of the wedge.

sions for the non-uniform currents are not evaluated but except the waves that are radiated by the fringe currents are derived according to the formula of

$$u_f = u_r - u_{PO} \tag{1}$$

where the function of u represents a component of the electromagnetic field. u_r is the rigorous value of the field, found from the solution of Sommerfeld [14]. u_{PO} is the PO field. u_f is the wave, which is radiated by the non-uniform currents. In this study we will use the MTPO integrals for u_r. u_{PO} will be represented by the PO integrals. The soft and hard surfaces will be taken into account. The soft surface is equivalent to a PEC surface where the electric field component is parallel to the edge. The hard surface expresses also a PEC surface for the magnetic field is parallel to the edge contour. In order to simplify the derivations, we will use scalar fields considering the boundary condition of the wedge for soft and hard surfaces as in acoustics.

2.1. Fringe waves for soft surface

A soft wedge is illuminated by an arbitrary incident wave of $u_i(P)$ as shown in Fig. 1. P is the observation point. The total field is equal to zero on the soft surface. The edge contour of the wedge is at $z \in (-\infty, \infty)$.

ψ is the outer angle of the wedge. The angle of incidence is α. The surface integral of MTPO can be written as

$$u_r(P) = u_i(P) + \frac{jk}{2\pi}\iint_S u_i(Q)\left(\cos\alpha - \cos\beta\right) f_s\left(\alpha,\beta,n\right) \times \frac{\exp(-jkR)}{R}dS \tag{2}$$

for the total scattered fields [12,15]. k is the wave-number. $f_s(\alpha, \beta, n)$ can be given by

$$f_s\left(\alpha,\beta,n\right) = \frac{\sin\left(\pi/n\right)}{n}\left\{\frac{1}{\cos\left(\pi/n\right) - \cos\left[\left(\pi-\beta+\alpha\right)/n\right]} - \frac{1}{\cos\left(\pi/n\right) - \cos\left[\left(\pi-\beta-\alpha\right)/n\right]}\right\} \tag{3}$$

where n is equal to ψ/π. $u_i(Q)$ is the value of the incident field at the scattering point. The surface of S is defined as $S = \left\{x \in (0,\infty), y = 0, z \in (-\infty,\infty)\right\}$. R_1 is $\sqrt{R^2 + (z - z')^2}$. The z' part of the integral can be evaluated as in Ref. [11], which leads to the equation of

$$u_r(P) = u_i(P) + \frac{k\exp\left(j\pi/4\right)}{\sqrt{2\pi}}\int_0^\infty u_i(Q)\left(\cos\alpha - \cos\beta\right) f_s\left(\alpha,\beta,n\right)\frac{\exp(-jkR)}{\sqrt{kR}}dx'. \tag{4}$$

The PO integral can be written as

$$u_{PO}(P) = u_i(P) - \frac{k\exp\left(j\pi/4\right)}{\sqrt{2\pi}}\int_0^\infty u_i(Q)\sin\alpha\frac{\exp(-jkR)}{\sqrt{kR}}dx' \tag{5}$$

for the same case [16]. The line integral representation of fringe waves can be found as

$$u_{fs}(P) = \frac{k\exp\left(j\pi/4\right)}{\sqrt{2\pi}}\int_0^\infty u_i(Q)\left[\sin\alpha + \left(\cos\alpha - \cos\beta\right) f_s\left(\alpha,\beta,n\right)\right]\frac{\exp(-jkR)}{\sqrt{kR}}dx' \tag{6}$$

according to Eq. (1) for soft surface. As a second step, we will represent Eq. (6) in terms of the Fresnel integral for plane wave incidence by using the method, introduced in Ref. [17]. Eq. (6) can be rewritten as

$$u_{fs}(P) = \frac{k\exp\left(j\pi/4\right)}{\sqrt{2\pi}}\int_0^\infty \exp\left(jkx'\cos\alpha\right)\left[\left(\cos\alpha - \cos\beta\right) f_s\left(\alpha,\beta,n\right) + \sin\alpha\right]\frac{\exp(-jkR)}{\sqrt{kR}}dx' \tag{7}$$

for a unit amplitude plane wave of

$$u_i(P) = \exp\left[jk(x\cos\alpha + y\sin\alpha)\right] \tag{8}$$

The GO contributions of the MTPO and PO integrals are equal and eliminate each other. This proposal can be proven by showing that the stationary phase contribution of Eq. (7) is equal to zero. The angle of scattering (β) is equal to $\mp\alpha$ at the stationary phase point [11,18]. It is apparent that the integrand of Eq. (7) is equal to zero both of these values of the scattering angle. The only contribution comes from the edge point of the integral at $x' = 0$. The formula for the evaluation of the radiated fringe wave can be introduced as

$$\frac{\exp\left(j\pi/4\right)}{\sqrt{\pi}}\int_{\alpha_e}^\infty h(\alpha)\exp\left[jkg(\alpha)\right]d\alpha = -\exp\left[jkg(\alpha_s)\right]\frac{2\xi_e h(\alpha_e)}{kg'(\alpha_e)}\mathrm{sign}\left(\xi_e\right)F\left[\left|\xi_e\right|\right] \tag{9}$$

according to Reference [17]. α_e and α_s are the edge point and stationary phase values of α. We suppose that the first and higher order derivatives of the amplitude function of $h(\alpha)$ are equal to zero. This condition is fulfilled in surface diffraction and grazing incidence problems, but we take into account the case of the wedge diffraction. $sign(x)$ is the signum function, which is equal to one for $x > 0$ and -1 for $x < 0$. $F[x]$ is the Fresnel integral, which can be defined by the expression of

$$F[x] = \frac{\exp\left(j\pi/4\right)}{\sqrt{\pi}}\int_x^\infty \exp\left(-jt^2\right)dt. \tag{10}$$