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An approximate three-dimensional analytical propagation formula for Gaussian beams through a cube corner reflector

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ABSTRACT

Based on the idea that the propagation of Gaussian beams passing through a cube corner reflector can be approximately regarded as a propagation of Gaussian beams passing through a hexagon aperture which is the intersection of three parallelogram apertures, and by using the method of expanding the aperture function into a finite sum of complex Gaussian functions, an approximate analytical propagation formula of Gaussian beams propagating through a cube corner reflector is derived and illustrated with numerical examples. It is found that the aberrance of the intensity distribution pattern becomes obvious with the increase of the incidence angle and orientation angle, and the aberrance brought by the incidence angle is larger that that by the orientation angle at same values.

1. Introduction

Cube corner reflector has the property of retro-direction reflection, so it is widely used in laser communication [1], laser measurement and tracking [2–5], optics transformation [6], and so on. The efficient reflected area will decrease with the increase of the incidence angle and the orientation angle, then the beam quality at the return place will decline and a low measure precision will be gotten. Studies have been done to make clear the effects of the incident angle and orientation angle on the efficient reflected area and output polarization states by using the method of geometry optics theory [4,5]. However, the phenomenon of diffraction plays an important role in the propagation of beams through a cube corner reflector, so the diffractions should be studied. What's more, the design of laser systems requires appropriate evaluation of beam characteristics. However, little attention is paid to study the analytical propagation formula of light beams through the cube corner reflector, except a few investigations discussed the diffraction by using numerical methods or software which can not get analytical propagation formula [7–10]. This is because the efficient reflected area of beams passing through a cube corner is hexangular, and the study of analytical formula of beams through hexagon aperture is difficult.

In this paper, based on the idea that the propagation of Gaussian beams passing through a cube corner reflector can be approximately regarded as a propagation of Gaussian beams passing through a hexagon aperture which is the intersection of three parallelogram apertures, and based on the fact that the hard-edged rectangular aperture can be expanded into an approximate finite sum of complex Gaussian functions that is widely used in treating the diffraction phenomena of light beams through the finite hard-edged apertures [11–15], and the parallelogram aperture can be considered as the deformed shape of a rectangular aperture [11], an approximate analytical method for Gaussian beams propagating through a cube corner reflector is derived. The paper is organized as follows. In Section 2, an approximate closed-form field distribution for Gaussian beams through a hexagon aperture is derived and analyzed. In Section 3, detailed numerical results are presented to illustrate the diffraction distribution characteristics of Gaussian beams through a cube corner reflector. Finally, Section 4 summarizes the main results obtained in this paper.

2. Propagation equations

According to the reflected theory of cube corner reflector [5], the efficient reflected region is a positive hexagon shown in Fig. 1(a) when the incident beam is upright to the surface ABC. The triangle A′B′C′ is the out pupil corresponding to the triangle ABC, now their barycenters are all O(0, 0). When the incident beam is not upright to surface ABC, the barycenter of the out pupil A′B′C′ will has a departure, and the efficient reflected area will decrease. We assume the barycenter of surface A′B′C′ is O′(O_x, O_y), the length of AB is l, the efficient reflected region is a contorted hexagon shown in Fig. 1(b) when $0 \le O_x \le \sqrt{3}\, l/6$, $0 \le O_y \le l/3 + \sqrt{3}\, O_x/3$, and is a

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parallelogram shown in Fig. 1(c) when $\sqrt{3}\,l/6 \le O_x \le 2\sqrt{3}\,l/3 - \sqrt{3}\,O_y$, $l/3 + \sqrt{3}\,O_x/3 \le O_y \le 2l/3 - \sqrt{3}\,O_x/3$. Here, O_x is mainly dependent on the incidence angle of the input beam to the cube, and O_y is mainly dependent on the orientation angle which is the angle of the orientation of the projection of the input beam on surface ABC to the orientation of line OA. The function relations of O_x and O_y with the incidence angle and orientation angle are not discussed any more in this paper, they can be gotten in Ref. [5].

As a result, the propagation of Gaussian beams through a cube corner reflector can be predigested to a diffraction process at a hexagon aperture abcdef on general conditions. From Fig. 1(b) we know that the hexagon aperture abcdef is the intersection of three parallelogram apertures: AbA′e, aCdC′, and fB′cB. The three parallelogram apertures can be considered as the deformed shape of three rectangular apertures, their aperture regions are respectively surrounded by four straight lines which can be represented by

$$\Theta_1 \pounds \begin{cases} A'b: y = \dfrac{\sqrt{3}}{3}x + \dfrac{1}{3}l - \dfrac{\sqrt{3}}{3}O_x + O_y \\ Ae: y = \dfrac{\sqrt{3}}{3}x - \dfrac{1}{3}l \\ A'e: y = -\dfrac{\sqrt{3}}{3}x - \dfrac{1}{3}l + \dfrac{\sqrt{3}}{3}O_x + O_y \\ Ab: y = -\dfrac{\sqrt{3}}{3}x + \dfrac{1}{3}l \end{cases} \tag{1}$$

$$\Theta_2 \pounds \begin{cases} Cd: x = -\dfrac{\sqrt{3}}{6}l \\ C'a: x = \dfrac{\sqrt{3}}{6}l + O_x \\ C'd: y = -\dfrac{\sqrt{3}}{3}x - \dfrac{1}{3}l + \dfrac{\sqrt{3}}{3}O_x + O_y \\ Ca: y = -\dfrac{\sqrt{3}}{3}x + \dfrac{1}{3}l \end{cases} \tag{2}$$

$$\Theta_3 \pounds \begin{cases} Bc: x = -\dfrac{\sqrt{3}}{6}l \\ B'f: x = \dfrac{\sqrt{3}}{6}l + O_x \\ Bf: y = \dfrac{\sqrt{3}}{3}x - \dfrac{1}{3}l \\ B'c: y = \dfrac{\sqrt{3}}{3}x + \dfrac{1}{3}l - \dfrac{\sqrt{3}}{3}O_x + O_y \end{cases} \tag{3}$$

In this paper, the optical field of the incident Gaussian beam can be written as

$$E_0(x_0, y_0, 0) = \exp\left(-\frac{{x_0}^2 + {y_0}^2}{{w_0}^2}\right) \tag{4}$$

where ω_0 is the waist width, and x_0, y_0 is the transverse coordinates of the beam.

Assume the distance from the position of the waist of the Gaussian beam to the cube corner reflector is L. Similar to the inferential method of Ref. [15] and by using the Collins diffraction formula [16], the optical field at the front of the cube corner reflector of the Gaussian beam through the free space can be given by

$$E_1(x_1, y_1, z_1) = \frac{ik}{4{P_1}^2 L}\exp[-\left(\frac{ik}{2L} + \frac{k^2}{4{P_1}^2 L^2}\right)\left({x_1}^2 + {y_1}^2\right) \tag{5}$$

where $k = 2\pi/\lambda$ is the wave number, with λ representing the wavelength, and

$${P_1}^2 = \frac{1}{{w_0}^2} + \frac{ik}{2L} \tag{6}$$

Based on the Collins diffraction formula, the optical field of the Gaussian beam through aperture abcdef and spread a distance of L can be expressed as

$$\begin{aligned} E_2(x_2, y_2, z_2) &= \frac{ik}{2\pi L}\exp\left[-\frac{ik({x_2}^2 + {y_2}^2)}{2L}\right] \\ &\times \iint\limits_{abcdef} E_1(x_1, y_1)\exp\left\{-\frac{ik[({x_1}^2 + {y_1}^2) - 2(x_2x_1 + y_2y_1)]}{2L}\right\}dx_1dy_1 \\ &= \frac{ik}{6\pi L}\exp\left[-\frac{ik({x_2}^2 + {y_2}^2)}{2L}\right] \\ &\times \iint\limits_{\substack{\Theta_1+\Theta_2+\Theta_3 \\ -Ccb-A'dc-dBe \\ -eC'f-fAa-aB'b}} E_1(x_1, y_1)\exp\left\{-\frac{ik[({x_1}^2 + {y_1}^2) - 2(x_2x_1 + y_2y_1)]}{2L}\right\}dx_1dy_1 \end{aligned} \tag{7}$$

In order to get a close-formed diffraction formula, we assume the integral in the aperture of abcdef can be approximately equivalent to the summation of the integral in the apertures of Θ_1, Θ_2, Θ_3. Here, the integral times in the aperture abcdef is thrice of that in the triangle apertures of Ccb, A′dc, dBe, eC′f, fAa and aB′b, and the diffraction effect of every side of aperture abcdef is double of the other two sides of the six small triangle apertures. So the diffraction effect of the apertures of the six small triangle apertures can be approximately ignored under far-field conditions, and Eq. (7) becomes

$$\begin{aligned} E_2(x_2, y_2, z_2) &\approx \frac{ik}{6\pi L}\exp\left[-\frac{ik({x_2}^2 + {y_2}^2)}{2L}\right] \\ &\times \iint\limits_{\Theta_1+\Theta_2+\Theta_3} E_1(x_1, y_1)\exp\left\{-\frac{ik[({x_1}^2 + {y_1}^2) - 2(x_2x_1 + y_2y_1)]}{2L}\right\}dx_1dy_1 \end{aligned} \tag{8}$$

Now let us consider the integral in area Θ_1. For the integral convenience, by rotating the x-y coordinate with a clockwise angle of 60° to x'–y' coordinate, the aperture region Θ_1 can be transformed to

$$\Theta_1' \pounds \begin{cases} A'b: x' = -\dfrac{\sqrt{3}}{6}l + \dfrac{1}{2}O_x - \dfrac{\sqrt{3}}{2}O_y \\ Ae: x' = \dfrac{\sqrt{3}}{6}l \\ A'e: y' = \dfrac{\sqrt{3}}{3}x' - \dfrac{1}{3}l + \dfrac{\sqrt{3}}{3}O_x + O_y \\ Ab: y' = \dfrac{\sqrt{3}}{3}x' + \dfrac{1}{3}l \end{cases} \tag{9}$$

where

$$x' = \frac{1}{2}x - \frac{\sqrt{3}}{2}y, \quad y' = \frac{\sqrt{3}}{2}x + \frac{1}{2}y \tag{10}$$

Then we can get

$$\begin{aligned} E_{21}(x_2', y_2', z_2') = \frac{ik}{6\pi L}\exp\left[-\frac{ik}{2L}\left({x_2}'^2 + {y_2}'^2\right)\right] \\ \times \iint\limits_{\Theta_1'} E_1(x_1, y_1)\exp\left[-\frac{ik}{2L}\left({x_1}'^2 + {y_1}'^2\right.\right. \\ \left.\left. -2x_2'x_1' + y_2'y_1'\right)\right]dx_1'dy_1' \end{aligned} \tag{11}$$

The hard aperture functions of the three parallelogram apertures of Θ_1', Θ_2, Θ_3 can be written as

$$A_{ny}(y) = \begin{cases} 1 & \left|y - O_{ny}\right| \le r_{ny} \\ 0 & \left|y - O_{ny}\right| > r_{ny} \end{cases} \tag{12}$$