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Generalization of Carré equation

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ABSTRACT

The present work offers new equations for phase evaluation in measurements. Several phase shifting equations with an arbitrary but constant phase shift between captured intensity signs are proposed. The equations are similarly derived as the so-called Carré equation. The idea is to develop a generalization of Carré equation that is not restricted to four images. Errors and random noise in the images cannot be eliminated, but the uncertainty due to their effects can be reduced by increasing the number of observations. An experimental analysis of the mistakes of the technique was made, as well as a detailed analysis of mistakes of the measurement. The advantages of the proposed equation are its precision in the measures taken, speed of processing and the immunity to noise in signs and images.

1. Introduction

Phase shifting is an important technique in experimental mechanics [1–3]. Conventional phase shifting equations require phase shift amounts to be known; however, errors on phase shifts are common for the phase shift modulators in real applications, and such errors can further cause substantial errors in the determination of phase distributions. There are many potential error sources, which may affect the accuracy of the practical measurement, e.g. the phase shifting errors, detector nonlinearities, quantization errors, source stability, vibrations and air turbulence, and so on [4].

Currently, the phase shifting technique is the most widely used technique for evaluation of interference fields in many areas of science and engineering. Its principle is based on the evaluation of the phase values from several phase modulated measurements of the intensity of the interference field. It is necessary to carry out at least three phase shifted intensity measurements to determine unambiguously and very accurately, the phase at every point of the detector plane. The phase shifting technique offers fully automatic calculation of the phase difference between two coherent wave fields that interfere in the process. There are various phase shifting equations for phase calculation that differ on the number of phase steps, on phase shift values between captured intensity frames, and on their sensitivity to the influencing factors during practical measurements [4].

The general principle of most interferometric measurements is the following. Two light beams (reference and object) interfere after an interaction of the object beam with the measured object, i.e. the beam is transmitted or reflected by the object. The distribution of the intensity of the interference field is then detected, e.g. using a photographic film, CCD camera, etc. The phase difference between the reference and the object beam can be determined using the mentioned phase calculation techniques. The phase shifting technique is based on an evaluation of the phase of the interference signal using phase modulation of this interference signal [5].

The article shows new equations for phase evaluation in measurements with an arbitrary but constant phase shift between captured intensity signs. The new equations are similarly derived as the so-called Carré equation, but the new algorithms may use more than four

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signs. Random noise in the images cannot be eliminated, but the error due to their effects can be reduced by increasing the number of observations. The relevance and importance of the proposed equation are its precision in the measures taken and the immunity to noise in signs.

2. Theory of phase shifting technique

The fringe pattern is assumed to be a sinusoidal function and it is represented by intensity distribution $I(x,y)$. This function can be written in general form as:

$$I(x,y) = I_m(x,y) + I_a(x,y)\cos[\phi(x,y) + \delta] \tag{1}$$

where I_m is the background intensity variation, I_a is the modulation strength, $\phi(x,y)$ is the phase at origin and δ is the phase shift related to the origin [6].

The general theory of synchronous detection can be applied to discrete sampling procedure, with only a few sample points. There must be at least four signal measurements needed to determine the phase ϕ and the term δ. Phase shifting is the preferred technique whenever the external turbulence and mechanical conditions of the images remain constant over the time required to obtain the four phase shifted frames. Typically, the technique used in this experiment is called Carré equation [7]. By solving Eq. (1) above, the phase ϕ can be determined. The intensity distribution of fringe pattern in a pixel may be represented by gray level, which varies from 0 to 255. With Carré equation, the phase shift (δ) amount is treated as an unknown value. The equation uses four phase shifted images as

$$\left\{\begin{array}{l} I_1(x,y) = I_m(x,y) + I_a(x,y)\cos\left[\phi(x,y) - \dfrac{3\delta}{2}\right] \\ I_2(x,y) = I_m(x,y) + I_a(x,y)\cos\left[\phi(x,y) - \dfrac{\delta}{2}\right] \\ I_3(x,y) = I_m(x,y) + I_a(x,y)\cos\left[\phi(x,y) + \dfrac{\delta}{2}\right] \\ I_4(x,y) = I_m(x,y) + I_a(x,y)\cos\left[\phi(x,y) + \dfrac{3\delta}{2}\right] \end{array}\right\} \tag{2}$$

Assuming the phase shift is linear and does not change during the measurements, the phase at each point is determined as

$$\phi = \arctan\left\{\frac{\sqrt{[(I_1 - I_4) + (I_2 - I_3)][3(I_2 - I_3) - (I_1 - I_4)]}}{(I_2 + I_3) - (I_1 + I_4)}\right\} \tag{3}$$

Expanding Eq. (3), we obtain the Carré equation as:

$$\tan(\phi) = \frac{\sqrt{\left|\begin{array}{cccc} -I_1^2 & +2I_1I_2 & -2I_1I_3 & +2I_1I_4 \\ & +3I_2^2 & -6I_2I_3 & -2I_2I_4 \\ & & +3I_3^2 & +2I_3I_4 \\ & & & -I_4^2 \end{array}\right|}}{\left|-I_1 + I_2 + I_3 - I_4\right|} \tag{4}$$

or emphasizing only the matrix of coefficients of the numerator and the denominator:

$$\tan(\phi) = \frac{\sqrt{\left|\sum_{r=1}^{4}\sum_{s=r}^{4} n_{r,s} I_r I_s\right|}}{\left|\sum_{r=1}^{4} d_r I_r\right|} \left\{\begin{array}{l} Num = \begin{bmatrix} n_{1,1} & n_{1,2} & n_{1,3} & n_{1,4} \\ & n_{2,2} & n_{2,3} & n_{2,4} \\ & & n_{3,3} & n_{3,4} \\ & & & n_{4,4} \end{bmatrix}, \quad Den = [\,d_1 \quad d_2 \quad d_3 \quad d_4\,] \\ Num = \begin{bmatrix} -1 & 2 & -2 & 2 \\ & 3 & -6 & -2 \\ & & 3 & 2 \\ & & & -1 \end{bmatrix}, \quad Den = [\,-1 \quad 1 \quad 1 \quad -1\,] \end{array}\right. \tag{5}$$

Almost all the existing phase shifting equations are based on the assumption that the phase shift at all pixels of the intensity frame is equal and known. However, it may be very difficult to achieve this in practice. Phase measuring equations are more or less sensitive to some types of errors that can occur during measurements with images. The phase shift value is assumed unknown but constant in phase calculation equations, which are derived in this article. Consider now the constant but unknown phase shift value δ between recorded images of the intensity of the observed interference field.

Considering N phase shifted intensity measurements, we can write for the intensity distribution I_k at every point of k recorded phase shifted interference patterns.

$$I_k(x,y) = I_m(x,y) + I_a(x,y)\cos[\phi(x,y) + \left(\frac{2k - N - 1}{2}\right)\delta] \tag{6}$$

where $k = 1, \ldots, N$ and N being the number of frames.

In Novak [4], several five-step phase shifting equations insensitive to phase shift calibration are described, and a complex error analysis of these phase calculation equations is performed. The best five-step equation, Eq. (7), seems to be a very accurate and stable phase shifting