Optik 122 (2011) 739–742

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Self-consistently determining mean field used in rate equations of semiconductor lasers

Ruhai Dou*, Jianguo Chen, Pengfei Wang

Department of Optoelectronics, Sichuan University, Chengdu 610064, People's Republic of China

ARTICLE INFO

Article history:
Received 27 September 2009
Accepted 26 May 2010

Keywords:
Semiconductor lasers
Rate equations
Mean field

ABSTRACT

An analytical expression for the mean field has been derived from the traveling wave rate equations in terms of the geometric mean of the counter-propagating fields inside the cavity, and an implicit solution has been obtained for the geometric mean inside the laser. These lead to the establishment of the analytical relation between the mean field inside the cavity and the output field from the diode laser. It is hoped that this relation may provide a common reference for the evaluation of the injection coefficient when studies are made on the injected semiconductor lasers, whose nonlinear dynamic characteristics are critically dependent on the injection coefficient, with rate equations simplified by adopting the mean field approximation.

1. Introduction

At present, the rate equations (REs) based on the mean field approximation (MF) have been recognized as the fundamental equations used to study the nonlinear dynamic characteristics of injected semiconductor lasers [1]. With the aid of the MFREs, a rich variety of nonlinear dynamic features, such as bifurcation, chaos, etc. from the external cavity semiconductor laser system (ECLD) and master-slaver semiconductor laser system (MSLD) have been revealed [2–7]. In order to evaluate the impacts of injection on the characteristics of the ECLDs and MSLDs, a parameter of vital importance, named the injection coefficient in this work, has been introduced into the rate equations. Here, the injection coefficient is defined as the power ratio of the injected field to the mean field inside the diode laser, which receives the injection radiation either feedback from the external cavity or emitted from another independent source. A vast amount of research results have suggested that the nonlinear features of the injected semiconductor lasers (LDs) are critically dependent on the injection coefficient. For example, Tkach and Chraplyvy [8] have pointed out that the nonlinear features of an ECLD can be categorized into 5 regimes when the injection coefficient varies from 10^{-4} to 10^{-1}. Thus, it becomes apparent that the amplitude of the mean field inside the LD cavity should be accurately specified.

Generally, the output photon density S (in this work, we assume that the electric field E is normalized such that $|E|^2$ is equal to S) of the LD can be measured experimentally. After taking into consideration the reflectivity of the laser output mirror, the amplitude of field inside the laser cavity can be evaluated. Logically, however, this idea cannot be implemented for the case when the mean field approximation is adopted. In the MFREs, the field amplitude has been assumed to be independent on the position and the cavity mirror reflectivities have been turned into a distributed coefficient instead of a lumped parameters. Under these circumstances, different methods have been proposed to relate the output field to the inside-cavity field.

In Ref. [9], the authors have outlined two methods. One method involves, working out the photon numbers inside the cavity using the MFREs, at first, and then distributing the photons lost within the photon lifetime according to certain ratios which were obtained by resorting to the variation pictures of the counter-propagating waves inside the laser cavity. Another method assumes that the ratio between the feedback field from the external mirror to the field directly reflected at the output facet (with a reflectivity of R) facing the external reflector of the diode laser is proportional to $(1-R)/\sqrt{R}$. Obviously, the first method has contradicted the hypothesis of mean field and cannot provide a self-consistent result. In addition, it is questionable if the photon numbers inside the cavity is worked out without considering the position dependence of the stimulated emission arising from the interaction between the light and gain medium. The expression provided by the second method can be extracted by regarding the output facet of the diode as a mirror with a power reflectivity of R and comparing the field directly reflected from the facet with that passing the facet twice and striking the external mirror once. In other words, the second method implies that the reflected field at the output facet be the mean field inside the diode. Later, we will prove that, by using the traveling wave REs, the ratio of the output field to the geometric mean of the

* Corresponding author.
E-mail address: drh77510@163.com (R. Dou).

0030-4026/$ – see front matter © 2010 Elsevier GmbH. All rights reserved.
doi:10.1016/j.ijleo.2010.05.016

counter-propagating fields inside the cavity is equal to $(1-R)/\sqrt{R}$, although there may not be a large difference between the geometric mean and arithmetic mean.

In this work, we have adopted a lumped facet reflectivity and solved the traveling wave REs similar to those frequently used to characterize the doped fiber lasers, in which the field variation along the propagation direction has been retained [10–12]. As a result, a self-consistent and analytical relation between the mean field inside the cavity and the output field from the diode laser has been established. And certain issues of concern have also been discussed.

2. Determine parameters used in the MFREs

Among various parameters, we choose two basic quantities, i.e., the photon density S and carrier density N. In the steady state, the traveling wave REs can be cast as

$$\frac{\pm dS^{\pm}(z)}{dz}=[a\Gamma(N-N_0)-\alpha]S^{\pm} \tag{1}$$

$$\frac{M-N}{\tau}=av_g(N-N_0)(S^{+}+S^{-}) \tag{2}$$

where the superscripts "±" identify the traveling directions of the fields inside the laser cavity, a is the differential gain coefficient, Γ is the confinement factor (which means that the field has been averaged along the transversal plane), N_0 is the transparency carrier density, α is the loss coefficient, τ is the carrier lifetime, v_g is the light speed inside the diode, and M is named the pump carrier density in this work, which is defined by

$$M=\frac{I\tau}{eV} \tag{3}$$

where I is the bias current, e is the absolute value of an electron charge, V is the volume of the active layer. Generally, τ is dependent on the carrier density N. However, difference of N from the threshold carrier density N_{th} is very small if the LD is above threshold biased and we retain the assumption made in the MFREs used to study the nonlinear dynamics of the injected LDs, i.e., taking τ as a constant. Also, the contribution of the spontaneous emission to the oscillation mode is neglected in (1), as often adopted in studying the deterministic characteristics of nonlinear dynamic systems.

The boundary conditions pertain to (1) are

$$S^{+}(L)R_2=S^{-}(L) \tag{4a}$$

$$S^{-}(0)R_1=S^{+}(0) \tag{4b}$$

where L is the cavity length of the LD, R_1 and R_2 are the reflectivities of the diode facets locating at $z=0$ and L, respectively.

Subtraction of the two equations of (1), multiplied by S^+ and S^-, respectively, it can be shown that the product of S^+ and S^- is independent on z, and we can let

$$S^{+}(z)S^{-}(z)=S_0^2 \tag{5}$$

From (5), it can be understood that S_0 is the geometric mean of the counter-propagating fields inside the LD cavity. And we will show that S_0 is not equal to the arithmetic mean later. Combining (4a), (4b) and (5), it can be derived that

$$S^{+}(L)=\frac{S_0}{\sqrt{R_2}},\quad S^{+}(0)=S_0\sqrt{R_1} \tag{6a}$$

$$S^{-}(L)=S_0\sqrt{R_2},\quad S^{-}(0)=\frac{S_0}{\sqrt{R_1}} \tag{6b}$$

And the output photon density at $z=L$ is

$$S_{out}(L)=S^{+}(L)(1-R_2)=\frac{S_0(1-R_2)}{\sqrt{R_2}} \tag{7}$$

From (7), it can be realized that the proportion constant between the output light and the geometric mean S_0 is $(1-R_2)/\sqrt{R_2}$, which is similar to the factor used in the second method mentioned in Section 1.

Using the equation for S^+, it can be proved that

$$S^{+}(L)=S^{+}(0)\exp[a\Gamma\int_0^L N(z)dz-a\Gamma N_0L-\alpha L] \tag{8}$$

Inserting (6a) into (8). One can derive that

$$\bar{N}=N_0+\frac{\alpha-\ln(r_1r_2)/L}{a\Gamma} \tag{9}$$

where

$$\bar{N}=\int_0^L\frac{N(z)dz}{L} \tag{10}$$

Inspecting (9), it can be realized that the right hand side is the threshold carrier density N_{th} of the LD, frequently used in the MFREs. And (9) tells us that the mean carrier density inside the laser cavity is also equal to N_{th} for the above-threshold biased LD when traveling wave REs are employed.

Now, let us focus on the average photon density inside the laser cavity. Summing up the two equations of (1) and considering (2), it can be deduced that

$$\frac{dS^{+}}{dz}-\frac{dS^{-}}{dz}=\frac{\Gamma(M-N)}{\tau v_g}-\alpha(S^{+}+S^{-}) \tag{11}$$

Integrating the above equation from 0 to L, taking into account (6a), (6b) and (9), it can be obtained that

$$\bar{S}^{+}+\bar{S}^{-}=\frac{\Gamma(M-N_{th})L/(\tau v_g)-\eta S_0}{\alpha L} \tag{12}$$

where

$$\bar{S}^{\pm}=\int_0^L\frac{S^{\pm}(z)dz}{L} \tag{13a}$$

$$\eta=\frac{(1-\sqrt{R_1R_2})(\sqrt{R_1}+\sqrt{R_2})}{\sqrt{R_1R_2}} \tag{13b}$$

Thus, we have derived relation between the mean field photon numbers and the geometric mean S_0. Once the quantity S_0 is obtained, the relation between the mean field inside the laser cavity and the output of the laser can be established. At this stage, we would like to point out that the above derivations can be extended to other quantities proportional to the square of the absolute value of the field, such as light intensity, radiation power, etc.

According to the physical picture adopted by the traveling wave REs, taking into consideration the phases of the counter-propagating fields inside the cavity and the definition of S^+ and S^- made in this work, the field inside the laser cavity can be put down as [9]:

$$E(z)=\sqrt{S^{+}}\exp(ikz)+\sqrt{S^{-}}\exp[ik(2L-z)] \tag{14}$$

where the quantity k is equal to $2\pi\mu/\lambda$, with μ and λ being the effective refractive index inside the diode and radiation wavelength, respectively. Assuming that $2kL$ is equal to an integer time of 2π for the self-regenerated mode and considering the fact that $\sqrt{S^+S^-}$ is a constant, an integration of (14) from $z=0$ to L, leads to

$$\int_0^L\frac{|E(z)|^2dz}{L}=\bar{S}^{+}+\bar{S}^{-}\equiv\bar{S} \tag{15}$$

In view of the above equation, the magnitude of the mean field used in MAREs should be equal to $\sqrt{\bar{S}}$, whose expression has been given in (12) already. From (12), it may be concluded that the quantity S_0