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Changes in the intensity distribution and polarization of non-uniformly polarized beams focused by a spherically aberrated lens with annular aperture

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Abstract

Based on the generalized Huygens–Fresnel diffraction integral and beam coherence polarization (BCP) matrix formulation, the focusing properties of a type of non-uniformly polarized (NUP) beam through a spherical aberrated lens with annular aperture are studied. The dependence of focal shift, intensity distribution, power in the bucket (PIB) and degree of polarization on the spherical aberration coefficient, obscure ratio and polarization parameters is illustrated numerically. The focusing of NUP beams by a spherically aberrated lens with circular aperture is treated as a special case, where the inner radius of the aperture is equal to zero.

Keywords: Non-uniformly polarized (NUP) beam; Spherically aberrated lens; Annular aperture; Focusing property; Degree of polarization

1. Introduction

The focusing properties of optical beams have been studied extensively both theoretically and experimentally due to their importance in practical applications. It was shown [1] that, when a linearly polarized high-power beam propagates through a laser amplifier, the thermally induced birefringence results in different polarization states in different parts of the laser beam; this kind of beams is referred to as non-uniformly polarized (NUP) beams. The parametric characterization of NUP beams was analyzed in Refs. [2,3] and global beam shaping with NUP beams was proposed in Ref. [4]. The focal shift and three-dimensional intensity distribution of focused NUP beams were investigated in Refs. [5,6], where the ideal lens without aberration was assumed. However, in practice the aberration may affect the focusing properties of beams. The purpose of the present paper is to study changes in the intensity distribution and polarization of NUP beams focused by a spherically aberrated lens with annular aperture. Based on the generalized Huygens–Fresnel diffraction integral [7] and beam coherence polarization (BCP) matrix formulation [8], the expressions for the intensity distribution, power in the bucket (PIB) and degree of polarization of focused NUP beams are presented in Section 2. The focusing properties are illustrated by numerical examples in Sections 3 and 4. Finally, Section 5 summarizes the main results obtained in this paper.

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2. Theoretical formulation

As shown in Fig. 1, assume that a spherically aberrated lens with annular aperture is located at the $z=0$ plane. The outer and inner radii of the aperture are a and b, respectively, the focal length of the lens is f, and the spherical aberration is described by a phase factor $\exp(-\mathrm{i}kC_4\boldsymbol{r}^4)$ [9], where C_4 is the spherical aberration coefficient and k is the wave number related to the wave length λ by $k=2\pi/\lambda$. A type of NUP beams is incident upon the lens whose field distribution at the $z=0$ plane in the cylindrical coordinate system reads as [5]

$$\begin{cases} E_x(r)=E_0\left[\dfrac{1}{1+(hr^2)^n}\right]^{1/2} \\ E_y(r)=E_0\left[\dfrac{(hr^2)^n}{1+(hr^2)^n}\right]^{1/2} \end{cases} \quad (b\leqslant r\leqslant a), \tag{1}$$

where E_x and E_y denote fields at x and y directions, respectively, h and n are two parameters characterizing the polarization distribution across the beam and E_0 is a constant.

Following the treatment in Ref. [8], the BCP matrix at the $z=0$ plane is expressed as

$$J(\boldsymbol{r}_1,\boldsymbol{r}_2)=\begin{bmatrix} J_{xx}(\boldsymbol{r}_1,\boldsymbol{r}_2) & J_{xy}(\boldsymbol{r}_1,\boldsymbol{r}_2) \\ J_{yx}(\boldsymbol{r}_1,\boldsymbol{r}_2) & J_{yy}(\boldsymbol{r}_1,\boldsymbol{r}_2) \end{bmatrix}, \tag{2}$$

where

$$J_{xx}(\boldsymbol{r}_1,\boldsymbol{r}_2)=\langle E_x^*(\boldsymbol{r}_1)E_x(\boldsymbol{r}_2)\rangle, \tag{3a}$$

$$J_{yy}(\boldsymbol{r}_1,\boldsymbol{r}_2)=\langle E_y^*(\boldsymbol{r}_1)E_y(\boldsymbol{r}_2)\rangle, \tag{3b}$$

$$J_{xy}(\boldsymbol{r}_1,\boldsymbol{r}_2)=\gamma_0\langle E_x^*(\boldsymbol{r}_1)E_y(\boldsymbol{r}_2)\rangle=\boldsymbol{J}_{yx}^*(\boldsymbol{r}_2,\boldsymbol{r}_1), \tag{3c}$$

where $\boldsymbol{r}_1$, $\boldsymbol{r}_2$ denote two position vectors at the $z=0$ plane, * is the complex conjugate, γ_0 is the cross-correlation coefficient and $0\leqslant|\gamma_0|\leqslant 1$ [10]. In Eqs. (3a) and (3b), the time average was made.

From Eqs. (3a) to (3b), the intensity distribution $I(\boldsymbol{r})$ and degree of polarization $P(\boldsymbol{r})$ at the $z=0$ plane are written as [8]

$$I(\boldsymbol{r})=J_{xx}(\boldsymbol{r},\boldsymbol{r})+J_{yy}(\boldsymbol{r},\boldsymbol{r}), \tag{4}$$

$$P(\boldsymbol{r})=\sqrt{1-\frac{4\mathrm{Det}[J(\boldsymbol{r},\boldsymbol{r})]}{[\mathrm{Tr}[J(\boldsymbol{r},\boldsymbol{r})]]^2}}, \tag{5}$$

where Det and Tr stand for the determinant and trace of the BCP matrix, respectively.

It follows from Eqs. (1)–(4) that the intensity distribution and degree of polarization at the $z=0$ plane are given by

$$I(r)=|E_x(r)|^2+|E_y(r)|^2=|E_0|^2, \tag{6}$$

$$P(r)=\left\{1-(1-|\gamma_0|^2)\frac{4(hr^2)^n}{[1+(hr^2)^n]^2}\right\}^{1/2}. \tag{7}$$

Eq. (6) indicates that the intensity at the $z=0$ plane shows a uniform distribution although $|E_x|^2$ and $|E_y|^2$ depend on the position, and the degree of polarization at the $z=0$ plane is dependent on the beam parameters n, h and γ_0, and is non-uniform in general. Only for $|\gamma_0|=1$, we have $P(r)=1$. For $|\gamma_0|<1$, $P(r)<1$, there is a partially polarized non-uniform beam at the $z=0$ plane. From $\mathrm{d}P(r)/\mathrm{d}r=0$ we obtain that at $r=\pm h^{-1/2}$ the degree of polarization reaches its minimum $P_{\min}=|\gamma_0|$ which is independent of the parameter n.

The propagation of NUP beams through a spherically aberrated lens with annular aperture is characterized by the generalized Huygens–Fresnel diffraction integral of the form [7]

$$E_\alpha(\boldsymbol{\rho},z)=\frac{\mathrm{i}}{\lambda B}\int E_\alpha(\boldsymbol{r},0)\exp[-\mathrm{i}kC_4\boldsymbol{r}^4] \times\exp\left[-\frac{\mathrm{i}k}{2B}(A\boldsymbol{r}^2-2\boldsymbol{\rho}\cdot\boldsymbol{r}+D\boldsymbol{\rho}^2)\right]\mathrm{d}^2\boldsymbol{r}, \tag{8}$$

where $\boldsymbol{\rho}$ is the position vector at z plane, $\alpha=x,y$ and the $ABCD$ matrix is

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}=\begin{pmatrix} -\dfrac{\Delta z}{f} & f+\Delta z \\ -\dfrac{1}{f} & 1 \end{pmatrix}, \tag{9}$$

with $\Delta z=z-f$.

The substitution from Eq. (9) into Eq. (8) and integration with respect to the azimuthal angle yield

$$E_\alpha(\eta,\Delta z)=\frac{\mathrm{i}2\pi a^2}{\lambda(f+\Delta z)}\exp\left[\mathrm{i}\frac{ka^2\eta^2}{2(f+\Delta z)}\right] \times\int_\varepsilon^1 E_\alpha(\xi,0)\exp[-\mathrm{i}kC_4a^4\xi^4] \times\exp\left[\frac{\mathrm{i}k}{2(f+\Delta z)}\frac{\Delta z}{f}a^2\xi^2\right]J_0\left(\frac{ka^2\eta\xi}{f+\Delta z}\right)\xi\,\mathrm{d}\xi, \tag{10}$$

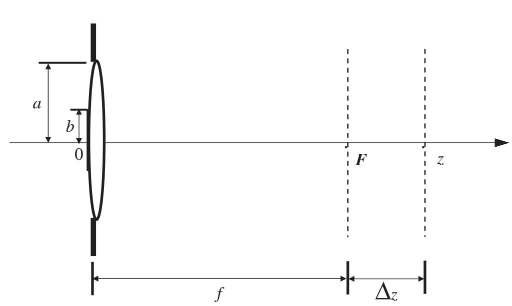

Fig. 1. Notation relating to the focusing of a non-uniformly polarized beam by a spherically aberrated lens with annular aperture.