Optik 122 (2011) 429–434

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Spectral properties of apertured stochastic electromagnetic twist anisotropic Gaussian Schell-model beam propagating in turbulent atmosphere

Haiyan Wang*, Xiangyin Li

Department of Physics, Nanjing University of Science and Technology, Nanjing 210094, PR China

ARTICLE INFO

Article history:
Received 11 August 2009
Accepted 28 February 2010

Keywords:
Stochastic electromagnetic beam
Turbulent atmosphere
Tensor method
Spectral properties

ABSTRACT

Based on the extended Huygens–Fresnel principle, analytical formulas are derived for the cross-spectral density matrix of an apertured stochastic electromagnetic twist anisotropic Gaussian Schell-model (ETAGSM) beam propagating in a turbulent atmosphere by use of a tensor method. Spectral properties of apertured ETAGSM beam are closely related with the strength of atmospheric turbulence, the aperture widths and the beam's parameters, etc. Our main attention was focused on the influence of the aperture widths, atmospheric turbulence, twist parameters and partial coherence on the spectral properties (including spectral degree of polarization, the spectral degree of coherence and the spectral density) of apertured ETAGSM beam propagating in turbulent atmosphere. Numerical calculation results and analysis are given.

1. Introduction

During the past several decades, with the help of the unified theory of coherence and polarization, the properties of partially coherent and partially polarized laser beams through the turbulent atmosphere have been investigated extensively. The changes in the degree of polarization of partially coherent electromagnetic beams propagating through atmospheric turbulence were studied by Wolf and colleagues. It was indicated that after sufficient propagation distance in turbulence, the degree of polarization of partially coherent electromagnetic beams tends towards the value that it has in the source plane, which is quite different from the behavior in free space [1–3]. In 2007, Du et al. pointed out that unlike an isotropic electromagnetic Gaussian Schell-model beam, the spectral degree of polarization of an anisotropic electromagnetic Gaussian Schell-model beam does not return to its value in the source plane after propagating at sufficiently large distance in the atmosphere [4].

However, the above-mentioned studies were restricted to the unapertured case. It is very important to study the propagation property of apertured laser beams, because the beam emitted from a laser system is more or less apertured in practice. As yet, only a few papers have dealt with the propagation of apertured laser beams in atmospheric turbulence, e.g., the propagation of plane waves, flattened Gaussian beams, cosh-Gaussian beams, and twist anisotropic Gaussian Schell-model beams diffracted by an aperture in turbulent atmosphere was studied in Refs. [5–8], respectively. However, to the best of our knowledge, the propagation properties of the apertured partially coherent and partially polarized electromagnetic twist anisotropic Gaussian Schell-model (ETAGSM) beam in turbulent atmosphere have never been studied. Based on the extended Huygens–Fresnel principle, we derive the electric cross-spectral density matrix and investigate the spectral properties (i.e., spectral density, the spectral degree of coherence and the spectral degree of polarization) of an apertured stochastic ETAGSM beam propagating in a turbulent atmosphere by a tensor method. Spectral properties of ETAGSM beam are closely related with the strength of atmospheric turbulence, the aperture widths and the beam's parameters, etc. Numerical calculation results and analysis are given.

2. Formulation

Consider partially coherent stochastic electromagnetic beam propagation close to the z-axis in the turbulent atmosphere, which can be characterized by a 2×2 electric cross-spectral density matrix [9], i.e.:

$$W(\tilde{\rho}, z; \omega) = \begin{bmatrix} W_{xx}(\tilde{\rho}, z; \omega) & W_{xy}(\tilde{\rho}, z; \omega) \\ W_{yx}(\tilde{\rho}, z; \omega) & W_{yy}(\tilde{\rho}, z; \omega) \end{bmatrix} \quad (1)$$

$$\text{where} \quad W_{ij}(\tilde{\rho}, z; \omega) = \left\langle E_i(\rho_1, z; \omega) E_j^*(\rho_2, z; \omega) \right\rangle \quad (i, j = x, y) \quad (2)$$

here $\tilde{\rho}^T = \left(\rho_1^T, \rho_2^T\right)$, ρ_1 and ρ_2 are position vectors of two arbitrary points in the receiver plane and T indicates the transpose. (*) denotes the complex conjugate, $\langle\ \rangle$ represents the average over an ensemble of realizations of the electric field, ω is the angular frequency, $E_x(\rho, z; \omega)$ and $E_y(\rho, z; \omega)$are electric field components in the receiver plane.

* Corresponding author. Tel.: +86 25 84303071.
E-mail address: njustwhy@mail.njust.edu.cn (H. Wang).

0030-4026/$ – see front matter © 2010 Elsevier GmbH. All rights reserved.
doi:10.1016/j.ijleo.2010.02.026

Based on the extended Huygens–Fresnel principle, the elements of the electric cross-spectral density matrix $W_{ij}(\tilde{\rho}, z;\omega)$ of an apertured stochastic ETAGSM beam in the turbulent atmosphere can be expressed as [10,11]:

$$W_{ij}(\tilde{\rho}, z;\omega) = \frac{k^2}{4\pi^2 z^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} W_{sij}(\tilde{r}, 0;\omega)H(r_1)H^*(r_2) \times \exp\left[-\frac{ik}{2z}(r_1-\rho_1)^2 + \frac{ik}{2z}(r_2-\rho_2)^2\right] \times \left\langle \exp[\psi^*(r_1,\rho_1,z;\omega)+\psi(r_2,\rho_2,z;\omega)]\right\rangle_m dr_1 dr_2 \quad (3)$$

with $\tilde{r}^T = (r_1^T, r_2^T)$, r_1 and r_2 are position vectors of two arbitrary points in the source plane. $k = 2\pi/\lambda = \omega/c$ denotes the wave number with λ being the wavelength and c is the wave speed in vacuum. $W_{sij}(\tilde{r}, 0;\omega)$ is the cross-spectral density in the source plane, $H(r_1)$ is the hard-aperture function. $\langle\,\rangle_m$ denotes averaging over the ensemble of turbulent media and can be expressed as [12,13]:

$$\left\langle \exp[\psi^*(r_1,\rho_1,z;\omega)+\psi(r_2,\rho_2,z;\omega)]\right\rangle_m = \exp\left\{-\frac{[(r_1-r_2)^2+(r_1-r_2)(\rho_1-\rho_2)+(\rho_1-\rho_2)^2]}{\rho_0^2}\right\} \quad (4)$$

here $\rho_0 = (0.545C_n^2k^2z)^{-3/5}$ is the coherence length of a spherical wave propagating in the turbulent atmosphere whose behavior is described by the Kolmogorov model and C_n^2 is the refractive index structure constant denoting the strength of turbulent atmosphere. In the derivation of Eq. (4), we have quadratic approximation for Rytov's phase structure function in order to obtain simpler and viewable analytical results [12,13]. This quadratic approximation has been shown to be reliable and has been widely investigated [4,14–16].

After some rearrangement, we can express Eq. (3) in the following tensor form:

$$W_{ij}(\tilde{\rho}, z;\omega) = \frac{k^2}{4\pi^2[\det(\tilde{B})]^{1/2}}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \times W_{sij}(\tilde{r}, 0;\omega)H(r_1)H^*(r_2) \times \exp\left[-\frac{ik}{2}(\tilde{r}^T\tilde{B}^{-1}\tilde{r} - 2\tilde{r}^T\tilde{B}^{-1}\tilde{\rho} + \tilde{\rho}^T\tilde{B}^{-1}\tilde{\rho}\right] \times \exp\left[-\frac{ik}{2}\tilde{r}^T\tilde{P}\tilde{r} - \frac{ik}{2}\tilde{r}^T\tilde{P}\tilde{\rho} - \frac{ik}{2}\tilde{\rho}^T\tilde{P}\tilde{\rho}\right] d\tilde{r} \quad (5)$$

with $\tilde{B} = \begin{pmatrix} zI & 0 \\ 0 & -zI \end{pmatrix}$, $\tilde{P} = \frac{2}{ik\rho_0^2}\begin{pmatrix} I & -I \\ -I & I \end{pmatrix}$, and I is a 2×2 matrix.

If the hard-edged aperture is circular and the radius is a_1, $H(r_1) = 1$ for $|r_1| \le a_1$ and $H(r_1) = 0$ for $|r_1| > a_1$. Then the hard-aperture function can be expanded as the following finite sum of complex Gaussian functions [17,18]:

$$H(r_1) = \sum_{m=1}^{M} A_m \exp\left(-\frac{B_m}{a_1^2}r_1^2\right) \quad (6)$$

with A_m and B_m are the expansion and the Gaussian coefficients; a table of A_m and B_m can be found in Ref. [17]. The simulation accuracy improves as M increases.

The element of the cross-spectral density matrix of an apertured stochastic ETAGSM beams at $z = 0$ can be expressed in the following tensor form [4]:

$$W_{sij}(\tilde{r}, 0;\omega) = C_iC_jD_{ij}\exp\left(-\frac{ik}{2}\tilde{r}^T M_{sij}^{-1}\tilde{r}\right) \quad (7)$$

here M_{sij}^{-1} is a transpose symmetric matrix and called partially coherent complex curvature tensor. For eTAGSM beams, the corresponding partially coherent complex curvature tensor M_{sij}^{-1} takes the following form [19,20]:

$$M_{sij}^{-1} = \begin{pmatrix} R_{sij}^{-1} + \left(-\frac{i}{2k}\right)\left(\sigma_{Isij}^2\right)^{-1} - \frac{i}{k}\left(\sigma_{gsij}^2\right)^{-1} & \frac{i}{k}\left(\sigma_{gsij}^2\right)^{-1} + u_{sij}J \\ \frac{i}{k}\left(\sigma_{gsij}^2\right)^{-1} + u_{sij}J^T & -R_{sij}^{-1} + \left(-\frac{i}{2k}\right)\left(\sigma_{Isij}^2\right)^{-1} - \frac{i}{k}\left(\sigma_{gsij}^2\right)^{-1} \end{pmatrix} \quad (8)$$

with R_{ij}^{-1} is a 2×2 wavefront curvature matrix and u_{sij} is a real-valued constant named the twist factor and $J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. σ_{Isij}^2 stands for transverse spot width matrix, σ_{gsij}^2 denotes transverse coherence width matrix. $\sigma_{Isij}^2, \sigma_{gsij}^2$ are all 2×2 matrices with transpose symmetry, given by

$$(\sigma_{Isij}^2)^{-1} = \begin{pmatrix} \sigma_{I11ij}^{-2} & \sigma_{I12ij}^{-2} \\ \sigma_{I21ij}^{-2} & \sigma_{I22ij}^{-2} \end{pmatrix} \quad (\sigma_{gsij}^2)^{-1} = \begin{pmatrix} \sigma_{g11ij}^{-2} & \sigma_{g12ij}^{-2} \\ \sigma_{g21ij}^{-2} & \sigma_{g22ij}^{-2} \end{pmatrix} \quad (9)$$

here the coefficients C_i, C_j and D_{ij} are independent of position but may depend on frequency [4]. Moreover, the factor D_{ij} has the properties [20]:

$$D_{ij} = 1 \quad \text{when} \quad i = j;\ \left|D_{ij}\right| \le 1 \quad \text{when} \quad i \ne j\ D_{ij} = D_{ij}^*. \quad (10)$$

After rearrangement, $W_{sij}(\tilde{r}, 0;\omega)H(r_1)H^*(r_2)$ in Eq. (5) can be expressed in the following tensor form:

$$W_{sij}(\tilde{r}, 0;\omega)H(r_1)H^*(r_2) = C_iC_jD_{ij}\sum_{m=1}^{M}\sum_{n=1}^{M}A_mA_n^* \times \exp\left[-\frac{ik}{2}\tilde{r}^T(B_{mn} + M_{sij}^{-1})\tilde{r}\right] \quad (11)$$

$$\text{where} \quad B_{mn} = \frac{2}{ika_1^2}\begin{pmatrix} B_mI & 0 \\ 0 & B_n^*I \end{pmatrix} \quad (12)$$

Then Eq. (5) is reduced to a sum of Collins-type integrals, and the input function for every Collins-type integral is $A_mA_n^* \times \exp\left(-\frac{ik}{2}\tilde{r}^T(B_{mn} + M_{sij}^{-1})\tilde{r}\right)$. After integration, we obtain:

$$W_{ij}(\tilde{\rho}, z;\omega) = C_iC_jD_{ij}\sum_{m=1}^{M}\sum_{n=1}^{M}A_mA_n^* \times \left(\det\left[\tilde{I} + \tilde{B}(B_{mn} + M_{sij}^{-1} + \tilde{P})\right]\right)^{-1/2} \times \exp\left[-\frac{ik}{2}\tilde{\rho}^T M_{ij}^{-1}\tilde{\rho}\right] \quad (13)$$

$$\text{here} \quad M_{ij}^{-1} = \tilde{P} + \tilde{B}^{-1} - \left(\tilde{B}^{-1} - \frac{1}{2}\tilde{P}\right)^T\left(B_{mn} + M_{sij}^{-1} + \tilde{B}^{-1} + \tilde{P}\right)^{-1} \times \left(\tilde{B}^{-1} - \frac{1}{2}\tilde{P}\right) \quad (14)$$