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Effects of third-order aberrations on the irradiance of self-image

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Abstract

We examine the effects of third-order aberrations exerted on the irradiance of image that is observable in a coherent self-imaging system. Both spherical aberration and astigmatism degrade the visibility of the image of a sinusoidal-type grating as well as blur the outline of the image of a rectangular-type grating. Coma laterally shifts the image of a sinusoidal-type grating on the image plane as well as changes a rectangular-type pattern into an asymmetrically blurred pattern. According to our analysis, the self-image of a high-density grating with a period of two times the optical wavelength is not at all affected by spherical aberration. In general a self-imaging system can always be corrected for astigmatism by shifting the image plane in its normal direction. We show that the self-image with defect can be well explained by taking the third-order aberrations and the focus-shift aberration into consideration.

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1. Introduction

Recently we have developed a geometrical theory of aberration for a self-imaging system that is very useful in evaluating the self-imaged patterns of linear, rectangular, or two-dimensional oblique periodicity [1–4]. When a coherent light is transmitted through (or reflected from) a periodic pattern, the incident light is split into several rays corresponding to a certain finite number of diffracted orders and the transmitted (or reflected) rays of different orders are combined again to make a self-image of the periodic pattern under appropriate conditions [5–13]. The self-imaging effect can be well described as the properties of the Fresnel field [14–18] that is generated on a quadric approximation of the optical path length in a Fresnel–Kirchhoff diffraction integral [19]. From the viewpoint of ray optics, the optical path of a self-imaging ray depends upon the terms of up to second order in the aperture coordinate, while that of the corresponding actual ray includes the higher-order terms in the aperture coordinate. The aberration of a self-imaging system results from the difference between the optical paths of the self-imaging ray and of the corresponding actual ray. In earlier works [1–3], we have analytically formulated the third (or fifth)-order aberration functions for a self-imaging system and then analyzed the role of the aberration functions in the amplitude spectrum of self-image. However, the effects of third (or fifth)-order aberrations on the irradiance of self-image have not been examined systematically. Though a self-imaging system is under coherent illumination, it is the irradiance of image that is observable. Therefore, it is of importance to analyze

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how the irradiance of image as an observable quantity is deformed (or blurred) by the aberrations.

In this paper, we examine the effects of third-order aberrations on the irradiance of self-image that is observable. First we formulate analytically the self-imaging field that is generated on a fourth-order approximation of the optical path length in a Fresnel–Kirchhoff diffraction integral [19] and then numerically evaluate the irradiance of the self-imaging field which suffers from third-order aberrations. We show that both spherical aberration and astigmatism are responsible for not only degrading the visibility of the image of a sinusoidal-type transmission grating but also both deforming the ripples in the image of a rectangular-type transmission grating and blurring the outline of the image. We also find that coma plays an important role in moving the image of a sinusoidal-type transmission grating in a direction parallel to the image plane, while the coma changes a rectangular-type transmission pattern into an asymmetrically blurred pattern. According to our analysis, the self-image of a high-density grating, of which the period is equal to two times the optical wavelength, is not at all affected by spherical aberration, and in general a self-imaging system can always be corrected for astigmatism by shifting the image plane in its normal direction. We also show that the defects in the self-images of an infinite pattern of bars and spaces can be well explained by our theoretical model in which we take the third-order aberrations and the focus-shift aberration into consideration. The theoretical model for a self-image with defect, discussed in this paper, will be useful in analyzing the blurred (or deformed) image of a high-density grating, which is generated by the oblique incidence of a coherent light.

2. Light disturbance for a self-image with third-order aberrations

We first consider a self-imaging system under coherent illumination as shown in Fig. 1. A monochromatic light of wavelength λ, emerging from a source point P, arrives at another point P' after being diffracted at some point Q in the opening of a grating. We choose the point P' at which the light disturbance appears like the opening in the grating. We take a Cartesian reference system with origin in the grating surface and with the x and y axes in the plane of the grating and choose the positive z-direction to the point into the half-space that contains the observation point P'. The coordinates of P and P' are denoted by (ξ, η, ζ) and (ξ', η', ζ') in source and image spaces, respectively, and the point Q has the coordinates $(x_Q, y_Q, 0)$ of which the dimensions are small compared in magnitude with ζ and ζ'. If the opening of the grating is parallel to the x-axis

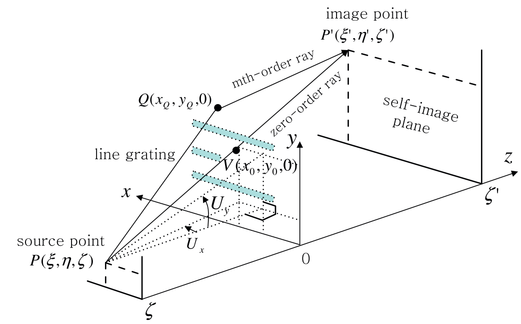

Fig. 1. Schematic diagram of a self-imaging system with coherent illumination. A monochromatic light of wavelength λ, coming from a source point P, illuminates a line grating of period p. The coordinate system is referred to the surface of the grating, parallel to the x and y axes but normal to the z-axis. The zero-order ray goes straight from the source point P to another point P' on the self-image plane and it also intersects the grating surface at the point V. The mth order diffracted ray passes from P to P' after being diffracted at some point Q in the aperture of the grating. ζ and ζ' denote the distances from the grating surface to the source and image points, respectively. We have the coordinates of (ξ, η, ζ) for P, $(x_0, y_0, 0)$ for V, $(x_Q, y_Q, 0)$ for Q, and (ξ', η', ζ') for P'.

and oscillating with a fundamental spatial period of p along the y-axis, its amplitude transmission at Q may be represented in a Fourier series of harmonic functions

$$\tau(y_Q) = \sum_{m=-\infty}^{\infty} b_m \exp\left(\mathrm{i}\frac{2\pi m}{p} y_Q\right), \tag{1}$$

where m is an integer and b_m stands for the amplitude of the harmonic wave with a spatial frequency of (m/p). When a coherent light of wavelength λ illuminates the grating of this type, the in-phase self-images of the grating are formed at distances ζ' which depend upon the source distance ζ and the grating constant p as

$$\frac{1}{\zeta'} - \frac{1}{\zeta} = \frac{\lambda}{2sp^2}, \tag{2}$$

with any positive integer s. In addition, the optical path of a self-imaging ray of mth order, propagated in air from a source point P through a point Q in the grating to an image point P', is determined by the raytracing equations

$$x_Q = x_0, \quad y_Q = y_0 - 2spm, \tag{3}$$

where the coordinates (x_0, y_0) denote the point V at which the zero-order ray (i.e., the straight line $\overline{PP'}$) intersects the surface of the grating. The above conditions (2) and (3) have been derived from the Fresnel–Kirchhoff diffraction formula in a quadric approximation of the optical path length and Fermat's principle [1–4,19].

To analyze the image of the grating suffering from third-order aberrations, however, we have to take into account the terms of up to fourth order in aperture