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Elliptic incoherent spatial solitons and their interactions in strongly nonlocal media with an anisotropic nonlocality

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Abstract

We investigate the propagation and interaction of elliptic incoherent spatial solitons (EISS) in strongly nonlocal *kerr* media with an anisotropic nonlocality based on the coherent density approach. An exact analytical solution of such EISS is obtained; the results show that such EISS can form with both isotropic and anisotropic coherence. Moreover, we find that the interaction properties of EISS are very similar to that of their coherent counterpart. Some numerical examples are presented and pertinent physics features are addressed.

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1. Introduction

Spatial solitons have drawn considerable attention in the past decades for their potential applications in the signal process and optical communications. Especially since the first observation of incoherent spatial solitons in photorefractive crystals [1,2], several theoretical and experimental investigations have been carried out [3–10]. More recently, incoherent spatial solitons were extended to nonlinear period lattices [11,12] and nonlocal nonlinear media [13–20]. Nonlocal incoherent spatial solitons are partially coherent beams and white-light beams propagating without changing their shape in nonlocal media. In general, the nonlocal response function plays an important role in the formation of $(2+1)$D nonlocal coherent (incoherent) solitons, which are discussed in Refs. [21–23]. Just as Refs. [8,9,22,23] pointed out, local, spatially isotropic nonlinear media cannot support coherent elliptic solitons, because such a local, isotropic nonlinearity cannot compensate for two different diffraction angles simultaneously. In contrast, elliptic incoherent beams with an anisotropic correlation can form solitons in isotropic media, while in anisotropic nonlocal media, anisotropic boundary conditions can support coherent elliptic solitons [22]. Hence, whether an anisotropic nonlocal nonlinearity may facilitate an elliptic incoherent soliton still remains a question, so as their interactions.

In this letter, we consider a nonlocal *kerr* medium which has an anisotropic nonlocality (for example, the nonlocal response function is Gaussian type with an elliptically nonlocal response), and then investigate the propagation and interaction of elliptic incoherent spatial solitons in such media. The results show that EISS can form with both isotropic and anisotropic coherence.

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Moreover, we find that some interaction properties of EISS are very similar to that in the (1 + 1)D case which we investigated in Ref. [5], and are very similar to that of their coherent counterpart, while they are different from that in saturable nonlinear media [9].

2. Theoretical model

To start, let us assume that the light beam propagates along the z-axis and diffracts both in the x and the y directions. Propagation of a two-dimensional quasi-monochromatic partially incoherent beam is governed by the following nonlinear Schrödinger equation [3,8]:

$$i\left(\frac{\partial f}{\partial z}+\theta_x\frac{\partial f}{\partial x}+\theta_y\frac{\partial f}{\partial y}\right)+\frac{1}{2k}\left(\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}\right)+\frac{k}{n_0}\delta n(I(x,y,z))f=0 \tag{1}$$

$$I(x,y,z)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|f(x,y,\theta_x,\theta_y,z)|^2\,\mathrm{d}\theta_x\,\mathrm{d}\theta_y \tag{2}$$

Eq. (1) describes partially incoherent beam propagation along the z direction of a non-instantaneous nonlinear medium, which is the evolution equation of the coherent density function f. θ_x (or θ_y) is an angle (in radians) with respect to the z-axis in the x (or y) direction. $k=n_0k_0, k_0=2\pi/\lambda$. $I(x,y,z)$ denotes the time-averaged total intensity. Here, we consider a strongly nonlocal *kerr* medium, in which the refractive index change $\delta n(I(x,y,z))$ induced by a beam with intensity $I(x,y,z)$ can be represented in a general form as [14]

$$\delta n(I(x,y,z))=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}R(x-x',y-y')I(x',y',z)\,\mathrm{d}x'\,\mathrm{d}y' \tag{3}$$

where $R(x,y)$ is the so-called nonlocal response function. In strongly nonlocal media, the nonlocal response function $R(x-x',y-y')$ can be expanded in Tailor's series with respect to x' about $x'=x$ and y' about $y'=y$ to second order, respectively. Following Refs. [24,25], we can rewrite Eq. (3) as

$$\delta n(I(x,y,z))=\frac{1}{2}P_0(R_x''(0)x^2+R_y''(0)y^2)+R(0)P_0+\frac{1}{2}R_x''(0)\int I(x',z)x'^2\,\mathrm{d}x'+\frac{1}{2}R_y''(0)\int I(x',y',z)y'^2\,\mathrm{d}y' \tag{4}$$

where $P=\iint I(x,y,z)\,\mathrm{d}x\,\mathrm{d}y$ is the beam power and P_0 is the beam power at $z=0$. $R(0)$ is the maximum of $R(x,y)$, $R_x''(0)=\mathrm{d}^2R(x,y)/\mathrm{d}x^2|_{x=0,y=0}$ and $R_y''(0)=\mathrm{d}^2R(x,y)/\mathrm{d}y^2|_{x=0,y=0}$, respectively. The last two terms of Eq. (4) can be neglected for the strong nonlocality; thus, we can rewrite the evolution equation (1) in another way:

$$i\left(\frac{\partial f}{\partial z}+\theta_x\frac{\partial f}{\partial x}+\theta_y\frac{\partial f}{\partial y}\right)+\frac{1}{2k}\left(\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}\right)+\frac{k}{n_0}\frac{1}{2}P_0(R_x''(0)x^2+R_y''(0)y^2)f=0 \tag{5}$$

and at $z=0$,

$$f(x,y,z=0,\theta)=\sqrt{r}\sqrt{G_N(\theta_x,\theta_y)}\phi_0(x,y) \tag{6}$$

where r is the intensity ratio, that is, $r=max(I)$, and $\phi_0(x,y)$ is the input spatial modulation function. $G_N(\theta_x,\theta_y)$ is the normalized angular power spectrum of the incoherent source. In general the incoherent angular power spectrum is Gaussian, i.e., $G_N(\theta_x,\theta_y)=\exp(-\theta_x^2/\theta_{0x}^2-\theta_y^2/\theta_{0y}^2)/\pi\theta_{0x}\theta_{0y}$, where θ_{0x} and θ_{0y} represent the width of the source angular power spectrum in the x and y directions, respectively. Less coherence means larger θ_{0x} and θ_{0y}. It is worth pointing out that Eq. (7) is very similar to the model discussed in Refs. [5,6], and so we believe that Eq. (7) also has an exact analytical solution of such an incoherent soliton. Following Refs. [5,6], we set $\phi_0(x,y,z=0)=\exp(-(x/x_0)^2-(y/y_0)^2/2)$ at the input, where x_0 and y_0 are the soliton widths of the incoherent soliton. In this case, the input power $P_0=\pi r^2x_0y_0$, and we can obtain the following relation [5]:

$$\begin{cases}\dfrac{-P_0R_x''(0)}{n_0}=\left(\dfrac{\theta_{0x}}{x_0}\right)^2+\left(\dfrac{1}{kx_0^2}\right)^2\\[2ex]\dfrac{-P_0R_y''(0)}{n_0}=\left(\dfrac{\theta_{0y}}{y_0}\right)^2+\left(\dfrac{1}{ky_0^2}\right)^2\end{cases} \tag{7}$$

From Eq. (7), we can conclude that an elliptic incoherent soliton can form with arbitrary incoherence properties, provided Eq. (7) is satisfied.

Furthermore, we can assume that the material response is a Gaussian function with an elliptically nonlocal response such that $\iint R(x,y)\,\mathrm{d}x\,\mathrm{d}y=1$, i.e.,

$$R(x,y)=\frac{1}{\pi\sigma_x\sigma_y}\exp\left(-\frac{x^2}{\sigma_x^2}-\frac{y^2}{\sigma_y^2}\right) \tag{8}$$

where σ_x and σ_y represent the extents of the nonlocality in the x and y directions, respectively, (in this letter, we consider only the case $\sigma_x\neq\sigma_y$, while the other case $\sigma_x=\sigma_y$ is discussed in Ref. [17] based on the mutual coherence function approach), which are much larger than the beam width in strongly nonlocal media. It is easy to obtain $R(0)=1/\pi\sigma_x\sigma_y$, $R_x''(0)=-2/\pi\sigma_x^3\sigma_y$ and $R_y''(0)=-2/\pi\sigma_y^3\sigma_x$. Combing Eq. (8) together with $R(0)$ and $R''(0)$, we can rewrite Eq. (4) as follows:

$$\delta n(I(x,y,z))=\frac{P_0}{\pi\sigma_x\sigma_y}\left(1-\frac{x^2}{\sigma_x^2}-\frac{y^2}{\sigma_y^2}\right) \tag{9}$$