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# The fluctuating phase error analysis in the digital grating phase-shifting profilometry

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ABSTRACT

The nonlinear response of the experimental system and the saturation of fringe patterns can induce the fluctuating phase error in the projection grating phase-shifting profilometry. Two major factors of the fluctuating phase error are discussed by simulation. The fluctuating phase error caused by the nonlinear response of the system is four times the frequency of the fringe pattern when the conventional four-frame phase extracting algorithm is used. However, such error can be decreased by five-frame algorithm. On the other hand, the fluctuating phase error caused by the fringe saturation is five times the frequency of the fringe pattern by using conventional five-frame phase extracting algorithm. A novel phase recovering algorithm is used to decrease the phase error caused by the saturation. Furthermore, the applicability range of the proposed phase recovering algorithm is analyzed by simulation and experiments with different saturation degree of the fringe pattern and nonlinearity of the measurement system.


## 1. Introduction

Many different techniques for quantitative phase measurement from fringe patterns have been developed in the past years [1–8]. The famous phase-shifting technique has become a standard in modern high-precision phase-measuring instruments. Compared to other methods, the digital grating phase-shifting method has the advantage of high accuracy and high measurement speed.

In the digital grating phase-shifting, periodic sinusoidal fringe patterns are projected onto the surface of the detected object by a LCD projector. Deformed patterns modulated by the surface are captured by a CCD camera. Generally, the CCD camera can present a straightforward and practical solution for recording the data from the measurement scene and when coupled with a frame grabber offer a convenient platform for further computer processing. However, the accuracy of the measurement system will be affected by the intensity error which includes the quantization error, the nonlinear response of the experimental system and the intensity saturation error. Several investigators have examined the sensitivity of phase-shifting algorithms to the intensity error such as quantization error [9] and detector nonlinearity [10,11]. However, few concerns are concentrated on the measurement phase error caused by the image intensity saturation. A novel phase recovering algorithm has been proposed to solve this issue [12,13]. But the applicability range and the nonlinear effects were not studied.

From the experimental results, fluctuating phase errors which are correlative with the spatial frequency of the fringe pattern have been found in the phase distribution map of the detected surface. In this paper, we deal especially with phase errors caused by the nonlinear response of the experimental system and the intensity saturation of captured fringe patterns. Characteristics of these phase errors are analyzed by simulation and experiment. The phase recovering algorithm corresponding to five-frame phase-shifting technique is deduced to decrease the phase error caused by the saturation of the fringe pattern. Moreover the applicability of the proposed phase recovering algorithm is discussed by simulations, and a specimen with more complex surface is detected to prove the feasibility of that method.

## 2. Fluctuating phase error analysis

### 2.1. Conventional phase extracting algorithm

In the digital projection grating phase-shifting profilometry, the fringe pattern intensity recorded by the CCD camera can be written as

$$I_i(x,y) = a + b\cos\left[\varphi(x,y) + \frac{2\pi i}{N}\right], \quad i = 0, 1, 2, \ldots, (N-1), \qquad (1)$$

where $I$ is the measured intensity, $a$ is the background intensity, $b$ is the intensity modulation amplitude, $\varphi$ is the phase to be analyzed, and $N$ is the number of the fringe patterns with phase-shifting.

Three unknowns $(a, b, \varphi)$ imply at least three pixels (data points) are needed to find the phase distribution. The conventional $N$-frame

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algorithms are simple to calculate. We can extract the phase value at each pixel independently in Eq. (1) as follows:

$$\varphi(x, y) = \arctan\left\{-\frac{\sum_{i=0}^{N-1} I_i(x, y)\sin(2i\pi/N)}{\sum_{i=0}^{N-1} I_i(x, y)\cos(2i\pi/N)}\right\}. \quad (2)$$

The major errors encountered in practical detections are the phase-shifting error, the intensity quantization error, nonlinear error and intensity saturation error. The intensity quantization error can be decreased by increasing the quantization level of the CCD camera [9]. As the phase-shifting of fringe patterns in this study is controlled by a computer program, the phase-shifting error can be ignored. Characteristics of the phase error caused by the nonlinearity and the saturation are discussed in following sections, respectively.

### 2.2. Nonlinear error analysis

The nonlinear error of the experimental system includes the nonlinear response of the LCD projector and the CCD camera. When the system nonlinearity of the second and third order are presented, the intensity in captured patterns take the form

$$I' = e_0 + e_1 I + e_2 I^2 + e_3 I^3, \quad (3)$$

where $I'$ is the detected intensity, $I$ is the incidence intensity, $e_0$, $e_1$, $e_2$ and $e_3$ each are the order response coefficients of the system, respectively.

In the simulative analysis of this paper, sinusoids are used to simulate one section of captured fringe patterns. We define $I_i(x) = 1 + \cos(0.1x + 2\pi i/N)$. The pitch of fringe patterns is 62.8, and fringe visibility denoted as $b/a$ is 100%. Moreover, the quantization level of the CCD detector is equal to $2^8$. By changing the mathematical approach, simulated signals with nonlinearity or saturation can be obtained.

A series of sinusoids presented as Eq. (4) are used to analyze the phase error introduced by the nonlinear response of the experimental system. $I_i(x) = 1 + \cos(0.1x + 2\pi i/N)$, where $N$ is equal to 4 and 5 for four-frame and five-frame phase-shifting, respectively. The response coefficients are $e_0 = 0$, $e_1 = 1$, $e_2 = -0.1$ and $e_3 = -0.2$. The nonlinear response curve is shown in Fig. 1(a). The phase information is extracted by the conventional four-frame and five-frame phase extracting algorithms, respectively. Simulative results of the phase error are shown Fig. 1(b).

$$I'_j(x) = \frac{255}{2}\{I_i(x) - 0.1[I_i(x)]^2 - 0.2[I_i(x)]^3\} \quad (4)$$

From simulative results, it is found that a fluctuating phase error will be obtained by using the conventional four-frame phase extracting algorithm, and the fluctuating error is four times the spatial frequency of fringes. However, the five-frame phase extracting algorithm can effectively decrease this fluctuating error.

### 2.3. Fringe saturation error analysis

Usually, fringe patterns captured by the CCD camera are quantized into $2^8$ levels. As the surface reflectivity and the incidence angle of the beam are different at the detected object surface, projected gratings modulated by the surface of the detected object are saturated in some of the bright fringe area. There will be an intensity saturation error between the incidence intensity and the detected intensity in the saturated area. Subsequently, phase error occurs when the phase is extracted by the conventional $N$-frame phase extracting algorithm with the inaccurate intensity of fringe patterns.

A series of saturated sinusoids shown in Fig. 2(a) is used to analyze the phase error caused by the intensity saturation in one

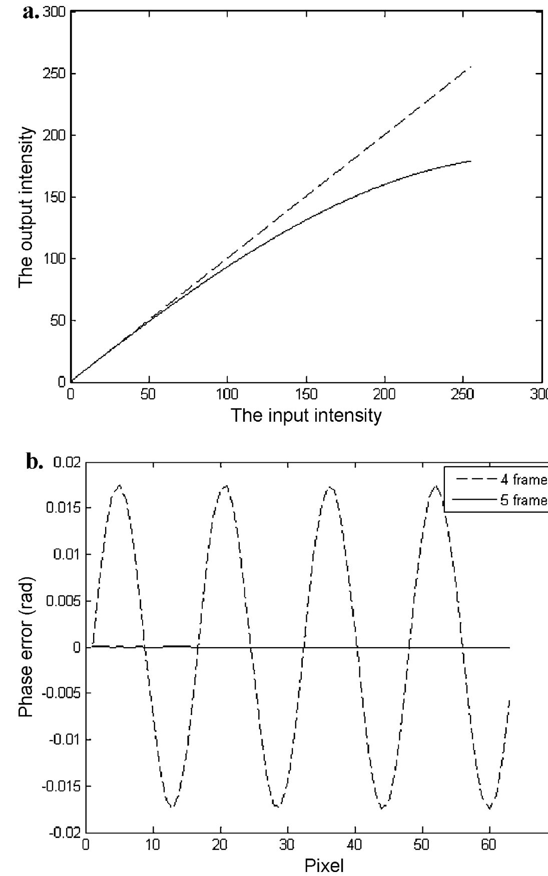


**Fig. 1.** Simulation of the phase error caused by nonlinearity: (a) nonlinear response curve; (b) phase error distribution.

period. The mathematical approach of fringe patterns is presented as Eq. (5), where the parameter '1.1' is used to control the saturation degree of fringe patterns, and intensity values which exceed the quantization range of the CCD camera are forcibly quantized as 255. $I_i(x) = 1 + \cos(0.1x + 2\pi i/5)$. The phase-shift between each signal is $2\pi/5$. The phase information is extracted by the conventional five-frame phase extracting algorithm. The phase error distribution is shown in Fig. 2(b).

$$I'_i(x) = \begin{cases} 255 & \text{for } 1.1 \times \frac{255}{2}[I_i(x)] > 255 \\ 1.1 \times \frac{255}{2}[I_i(x)] & \text{for } 1.1 \times \frac{255}{2}[I_i(x)] \le 255 \end{cases} \quad (5)$$

From Fig. 2(b), a fluctuating phase error is also presented along the fringe pattern. Moreover, the frequency of the fluctuating phase error is five times higher than the spatial frequency of the fringe pattern.

### 2.4. Height error analysis with different fringe periods

The phase error $\Delta\varphi$ depends on the phase $\varphi$ and the intensity error $\Delta I_i$. From the principle of the projected phase measurement, the relationship between the surface height error denoted as $\Delta h$ and the phase error are shown as follow:

$$\Delta h \propto \frac{p}{2\pi \tan\alpha}\Delta\varphi, \quad (6)$$

where $p$ is the pitch of the captured fringe pattern, $\alpha$ is the projection angle.

If the fringe pattern period is wide enough, the phase error will be significantly large than using shorter period of fringes. Hence the